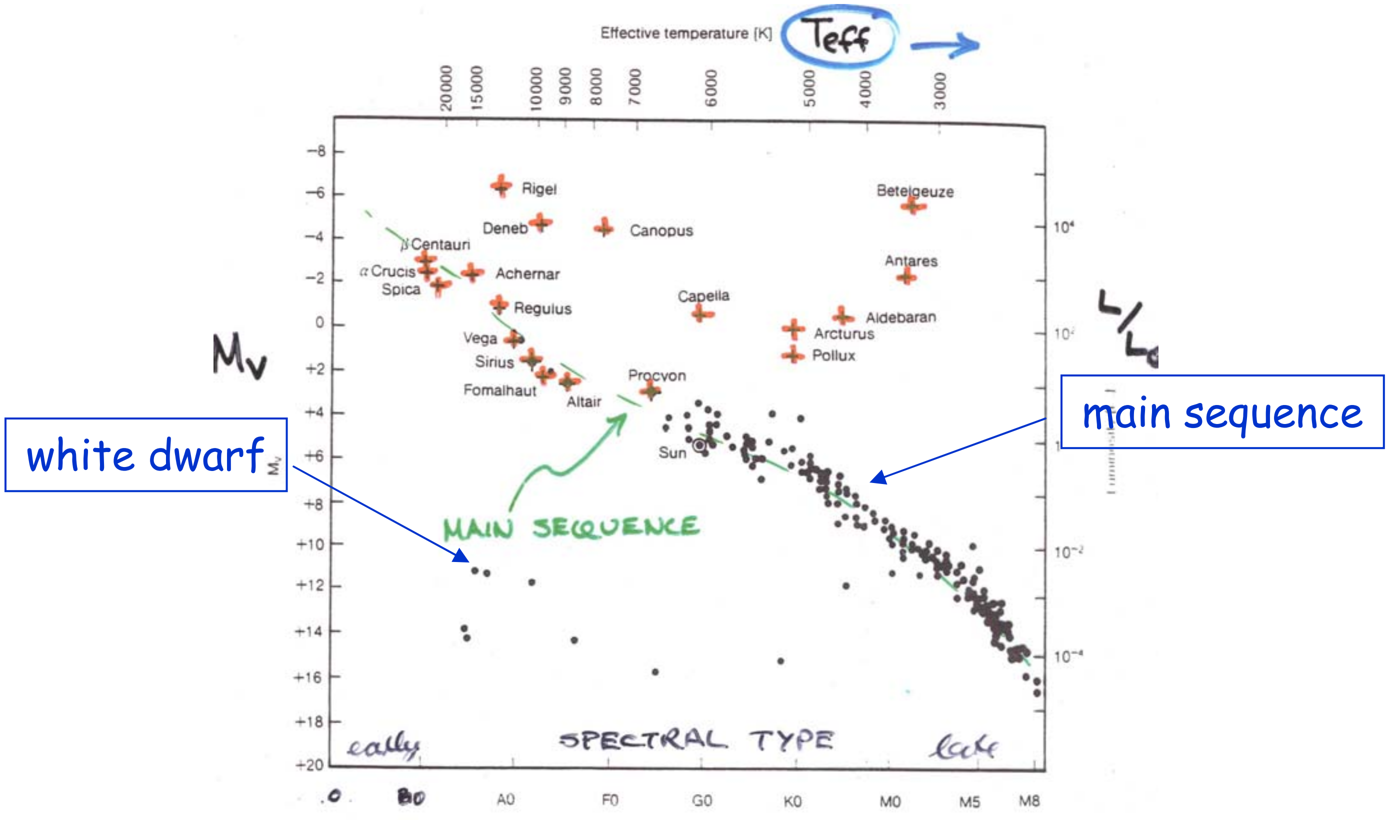
AY 20 Fall 2010

Hertzsprung-Russell Diagram & Stellar Radiation

Reading: Carroll & Ostlie, Chapter 8.2, Chapter 7

Hertzsprung-Russell Diagram



original form of diagram: M_v as function of spectral type

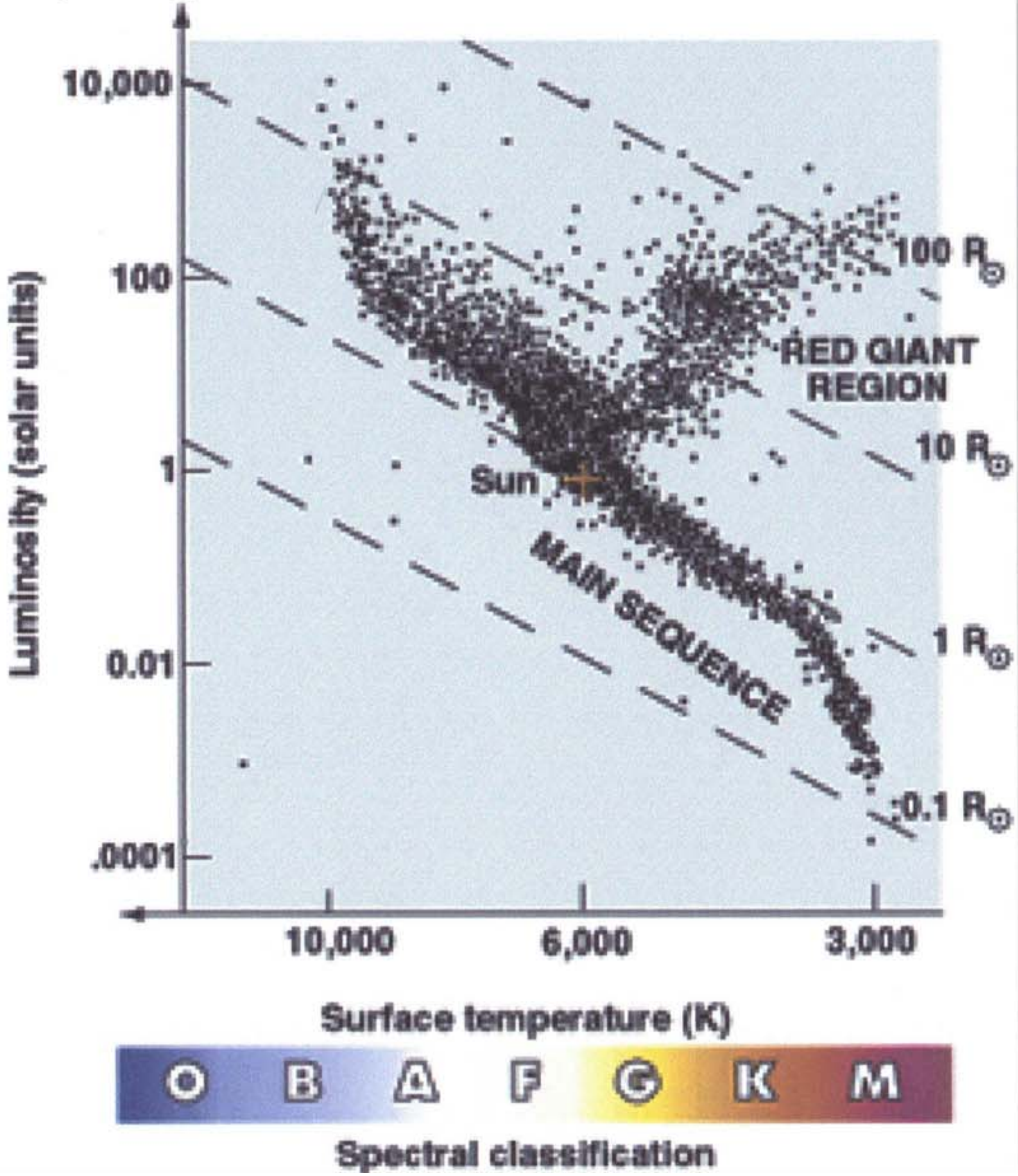
- ● stars within 1 kpc of Sun (solar "neighborhood) + brightest stars (apparent)

distribution of stars seems to have a pattern; 90% on main sequence

- ➢ HNR → main sequence stars – dwarfs; luminous, late spectral type - giants

M_v and spectral type (i.e. L and T_{eff}) are *intrinsic* stellar properties

H-R (Hertzsprung-Russell) diagram



stars are not spread over the entire range of L,T

⇒clue to the way stars work

Recall Yerkes (MKK) classification system Ia - V

$L_* = 4\pi R_*^2 F = 4\pi R_*^2 \sigma T_e^4$

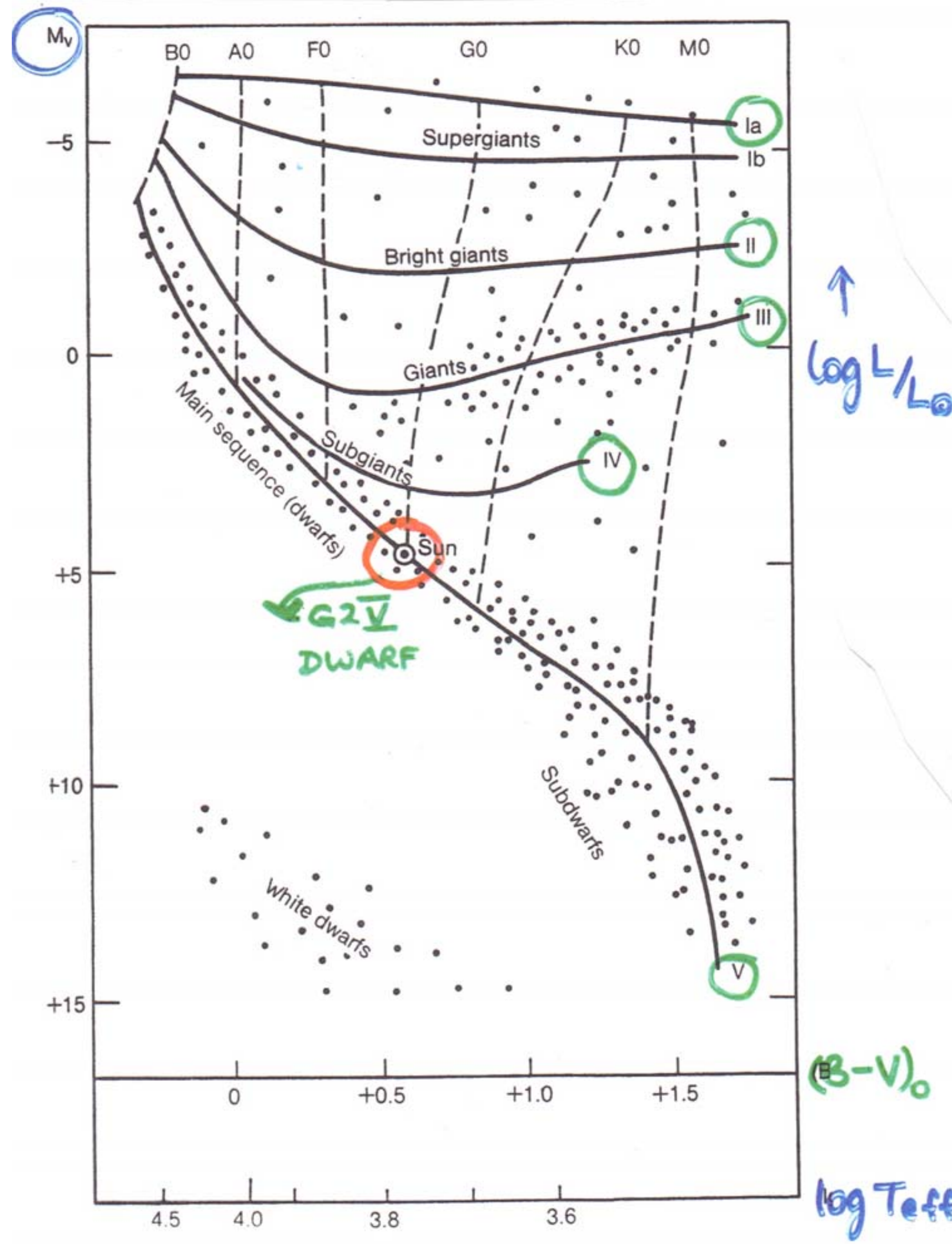
→ absolute magnitude of stars of same spectral type varies with R_*

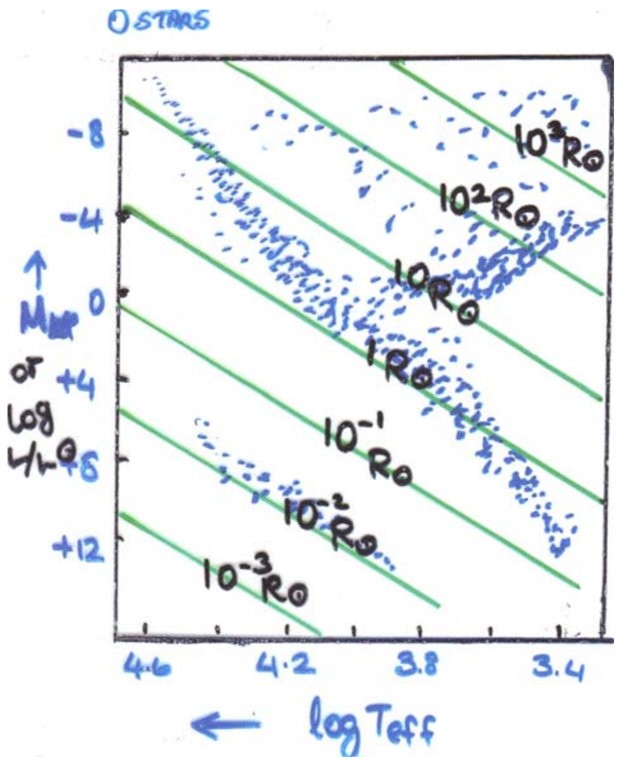
$$R_* = \frac{1}{T_{eff}^2}\sqrt{\frac{L}{4\pi\sigma}}$$

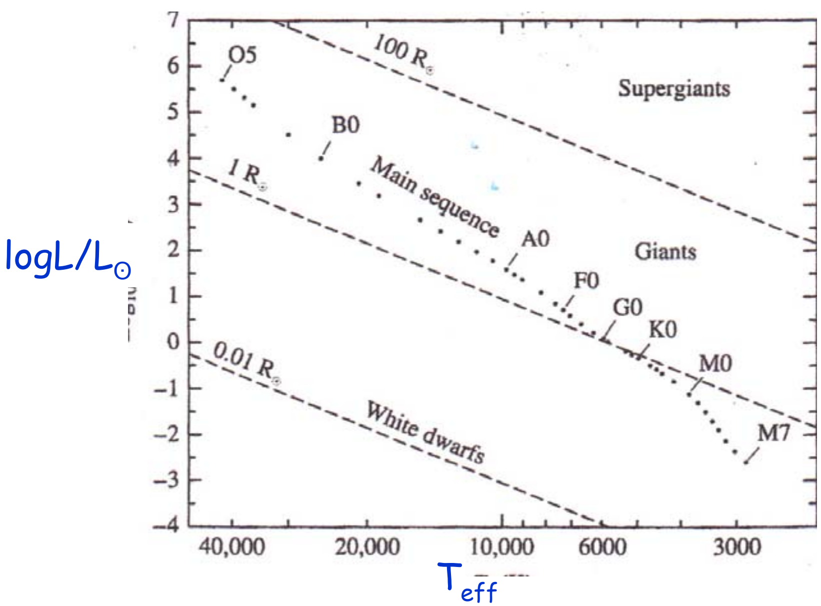
for fixed R_*, $\log L_* \propto \log T_{eff}$

→ lines of constant R_* in H-R diagram

Express R_*, T_*, L_*, in terms of $R_\odot$, $T_\odot$, $L_\odot$,





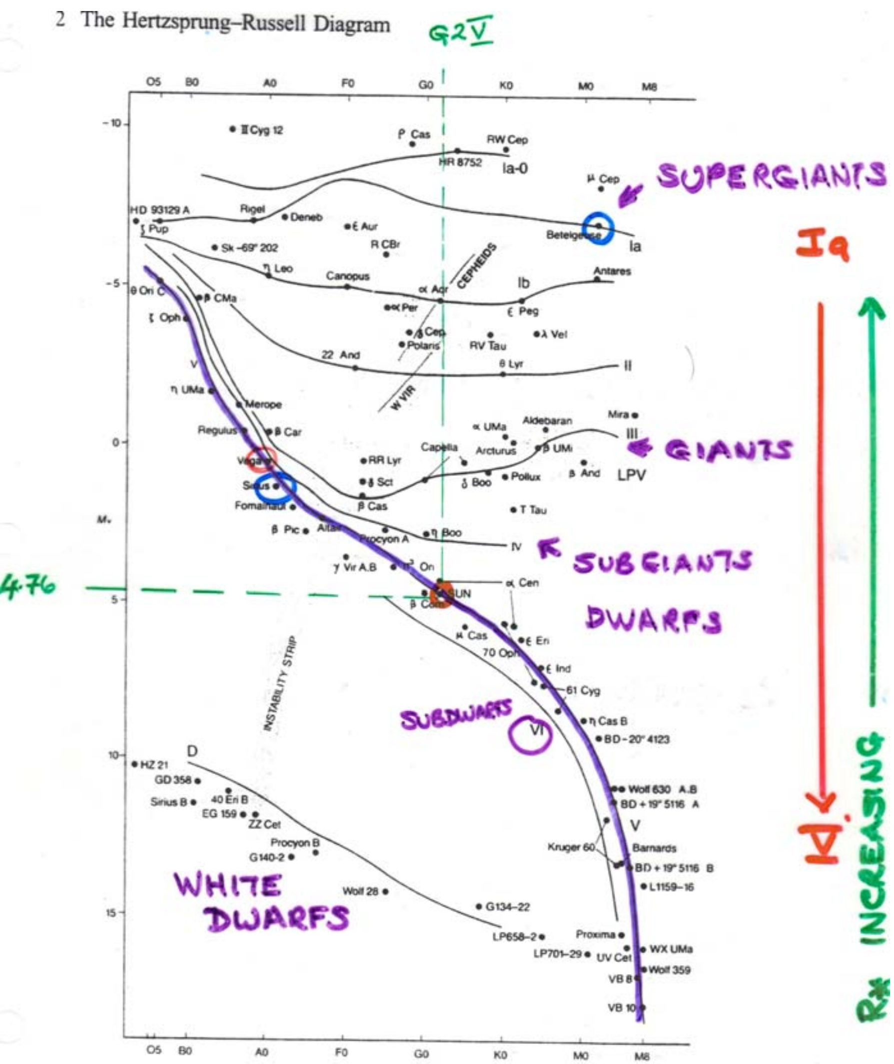


- Supergiants R > 100 $R_\odot$
- Giants 10 $R_\odot$ < R < 100 $R_\odot$
- main sequence stars (dwarfs)
- 0.1 $R_\odot$ < R < 20 $R_\odot$
- stellar density, $\rho_* = \frac{M_*}{\frac{4}{3}\pi R_*^3}$

varies with position in H-R diagram

- C&O 8.2.1: average densities of
  Sun (G2V), Sirius (A1V), Betelgeuse (~MIa)
  1.4 gms/cm^3 0.8 gms/cm^3 $10^{-8}\rho_\odot$
- Later: position of star on main-sequence depends on its mass

Stellar Properties from H-R diagram



ure 8.15 Luminosity classes on the H–R diagram. (Figure from Kaler, rs and Stellar Spectra, © Cambridge University Press 1989. Reprinted h the permission of Cambridge Universit

spectral type + luminosity class
(class indicates line width) → M_v

distance from m-M =5logd -5

spectroscopic parallax*

radii from L + T_{eff} (see page 5 here),
hence density if mass known

white dwarfs	supergiants
$R \sim 0.001\ R_\odot$	$R \sim$ few x $10^3\ R_\odot$
$\rho \sim 10^9\ kg/m^3$	$\rho \sim 10^{-4}\ kg/m^3$

(some radii from interferometric measures of angular diameters of stars with known parallax)

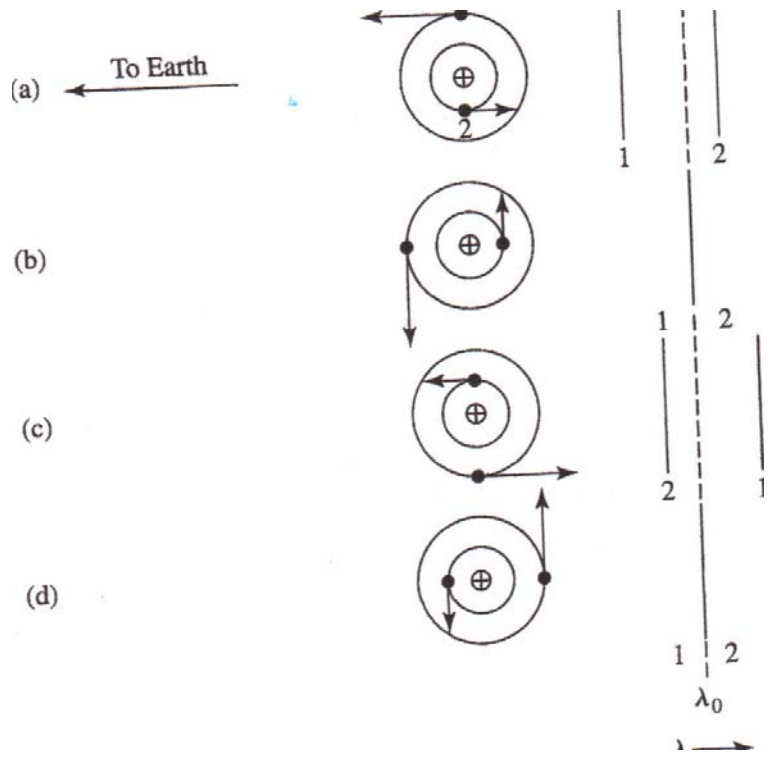
masses from binary star observations

Stellar parameters from binary system measures

- Binary or multiple star systems common
- Classifications
  - Optical doubles – neighbors by coincidence
  - Visual binaries – both members of system can be resolved
    - Can monitor motion if P not too long → angular sep^n from ctr of mass
  - Astrometric binaries – only one member visible; its motion reflects presence of a companion
  - Eclipsing binaries – orbital plane oriented so that one star periodically eclipses other → associated variations in light intensity
    - Light curves show two stars → relative T_{eff} for each star from depth of minima; radii based on eclipse duration
  - Spectrum binaries – two distinguishable spectra. Orbital period may be so long that no variation of wavelength with time seen

Spectroscopic binaries –periodic shift in positions of spectral lines is observed

- Lines shift around rest frequency as stars orbit each other
- Easiest to see when velocities in observer's line of sight
- Some component of velocity along line of sight required
- Orbital period should be not too long
- If one component much more luminous than other only one set of shifting lines



Kepler's Laws generalized for binary systems

Both objects in a binary orbit the center of mass in ellipses with the center of mass at one focus

Kepler's 2nd law becomes $dA/dt = \frac{1}{2} L/\mu = \text{constant}$,

L =total angular momentum of system; $\mu \equiv m_1 m_2/m_1 + m_2$ = reduced mass
$\equiv$ a fixed mass μ orbiting center of mass

Integrating for one orbital period: $t = P$, $A = \frac{1}{2} LP/\mu$

$A = \pi ab$ and $b^2 = a^2(1-e^2)$, $\quad \therefore (LP/2\mu)^2 = \pi^2 a^4(1-e^2)$

$$\therefore P^2 = 4\pi^2 \mu^2 a^4 (1-e^2)/L^2$$

From conservation of angular momentum:

$$L = \mu(GMa(1-e^2))^{1/2}$$

$$\therefore P^2 = 4\pi^2 a^3/G(m_1 + m_2)$$

P^2 inversely proportional to total mass of the system

With P in years, a in AU, solar masses, constant = 1

Stellar masses from visual binaries

For orbital plane perpendicular to line of sight & in center of mass reference frame,*

$m_1/m_2 = a_2/a_1$

a_2 and a_1 are semi major axes of 2 ellipses

Angles subtended by these axes a distance d, α_1 and α_2

$\alpha_1 \approx a_1/d$ and $a_2 \approx a_2/d$

$\therefore m_1/m_2 = \alpha_2/\alpha_1 \rightarrow$ mass ratio

From $P^2 = 4\pi^2 a^3/G(m_1 + m_2)$,

get $(m_1 + m_2)$ if semi-major axis of or of reduced mass ($a = a_1 + a_2$) knowr need d;

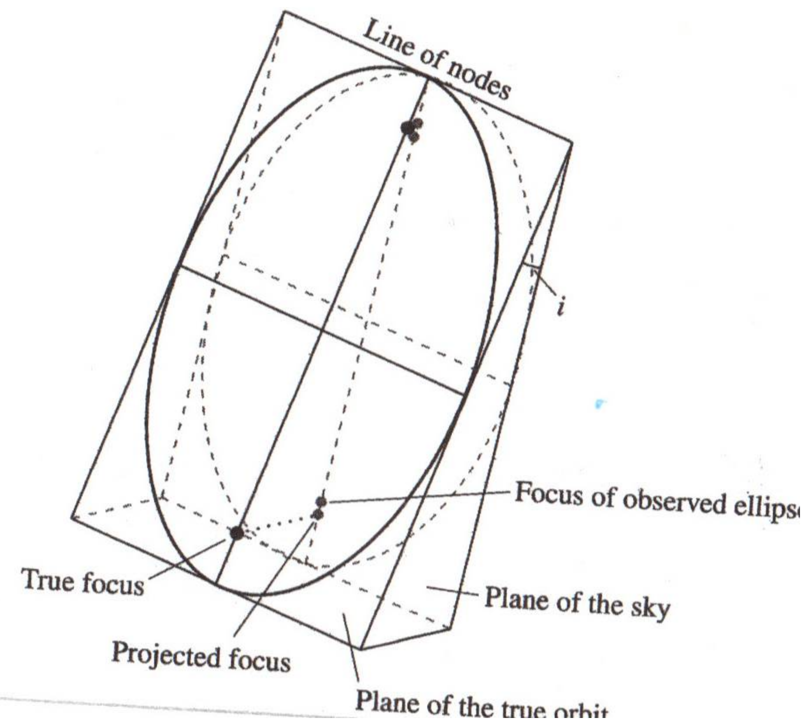
$(m_1 + m_2)$ and $m_1/m_2 \rightarrow$ masses

May have to take into account prope motion of C of M

Inclination of orbital plane also needed

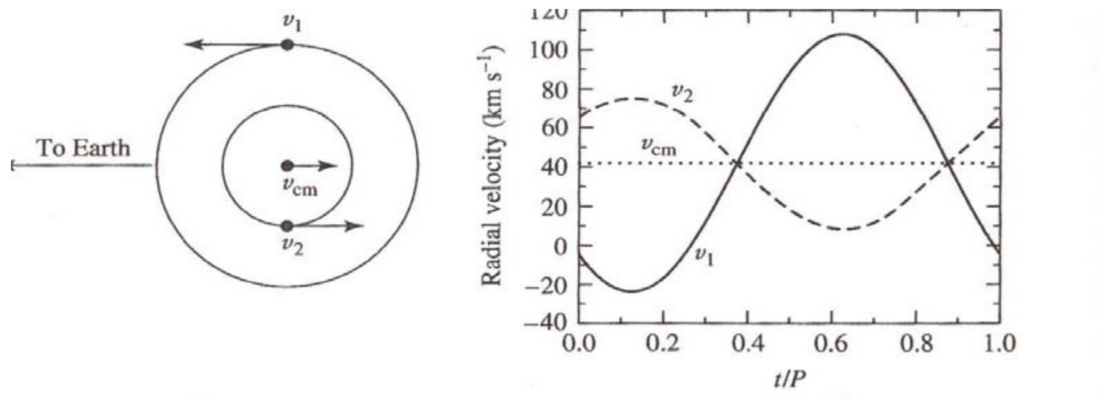
$m_1 + m_2 = = 4\pi^2 \alpha^3/G P^2 (d/\cos i)^3$

Mass Determination Using Visual Binaries



Masses from double-line spectroscopic binaries

Masses $1M_\odot$ and $2M_\odot$
P = 30d; v_{CofM} =42km/s



(a) (b)

Max observed radial velocities $v_r = v_1\sin i$, $v_2\sin i$; sinusoidal curves if i =90°; changing i merely changes amplitude of sinusoids

For e <<1, velocities constant $v_1 = 2\pi a_1/P$, $v_2 = 2\pi a_2/P$

$\therefore m_1/m_2 = v_2/v_1$; substituting $\rightarrow$ $m_1/m_2 = v_{2r}/v_{1r}$

ratio of masses independent of sini

From $a = a_1 + a_2 = P/2\pi \times (v_1 + v_2)$ and Kepler's 3rd:

$$m_1 + m_2 = P/2\pi G \times (v_1 + v_2)^3$$

$$\therefore m_1 + m_2 = P/2\pi G \times (v_{1r} + v_{2r})^3 / \sin^3 i$$

But have to be able to measure both radial velocities

Single line spectroscopic binary

- One star much more luminous than other
- e.g. planets orbiting other stars
- Replace v_{2r} in terms of stellar masses and v_{1r}

$$m_1 + m_2 = Pv^3_{1r}/2\pi G \sin^3 i \times (1 + m_1/m_2)^3$$

→ Mass Function Equation

$$\frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{v_1^3 P}{2\pi G}$$

Mass function → only lower limit to mass of m_2 if sini unknown

Mass Luminosity Relation

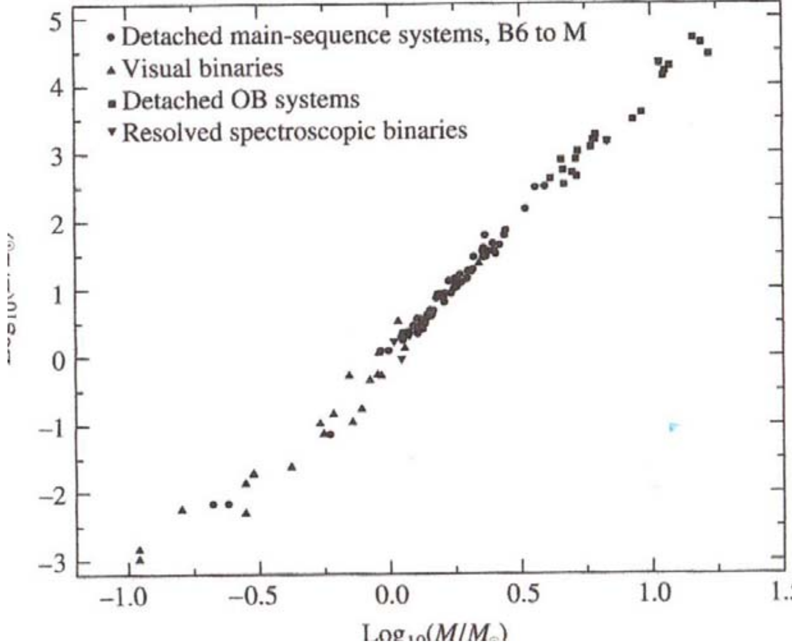
Masses of binary stars define empirical mass-luminosity relation

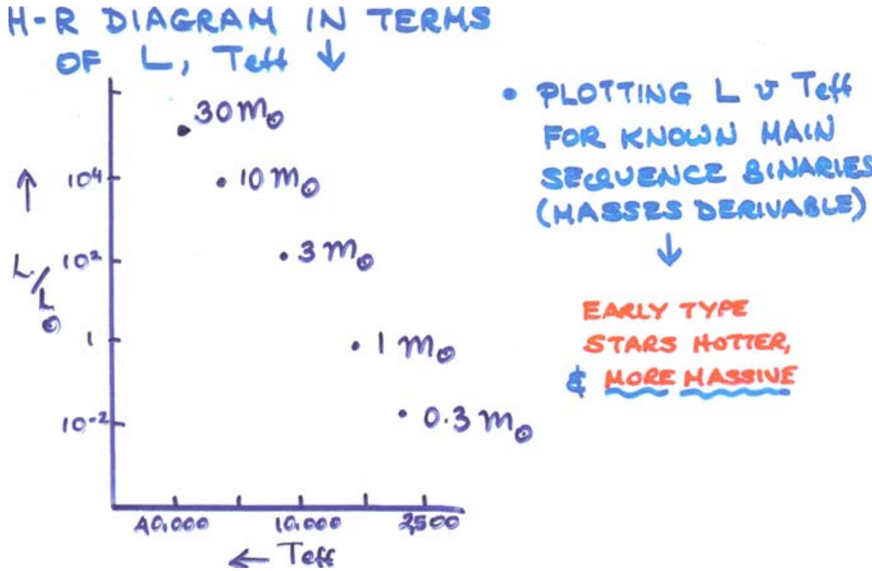
For $M > 3\ M_{\odot}$, $L \propto M_*^{3}$

For $M < 0.5\ M_{\odot}$, $L \propto M_*^{2.5}$

VERY approximate

Plotting stars of known mass on H-R diagram (L v. T_{eff}) also instructive





Stellar properties from observables

Direct measurements

- Distances (parallax)
- Luminosities (U,B,V etc)
- Masses (binaries)
- Radii (eclipsing binaries, interferometeric measures)

Spectra also provide info on:

- Rotational velocities
- Chemical abundances
- Magnetic fields
- Mass inflow-outflow

Using stellar spectra:

- Spectral type ≡ T_{eff}
- Luminosity classes ≡ gravity, pressure, density
- Radial velocities, z

Position on HR Diagram:

- Stellar radii
- Distances (spectroscopic parallax)
- main-sequence masses (approx)
- Ages of stars (later)