AY 20 Fall 2010

Electromagnetic Radiation:

Understanding Stellar Spectra
&
Hertzsprung-Russell Diagram

Reading: Carroll & Ostlie, Chapter 8

Spectral Types of Stars → temperature sequence

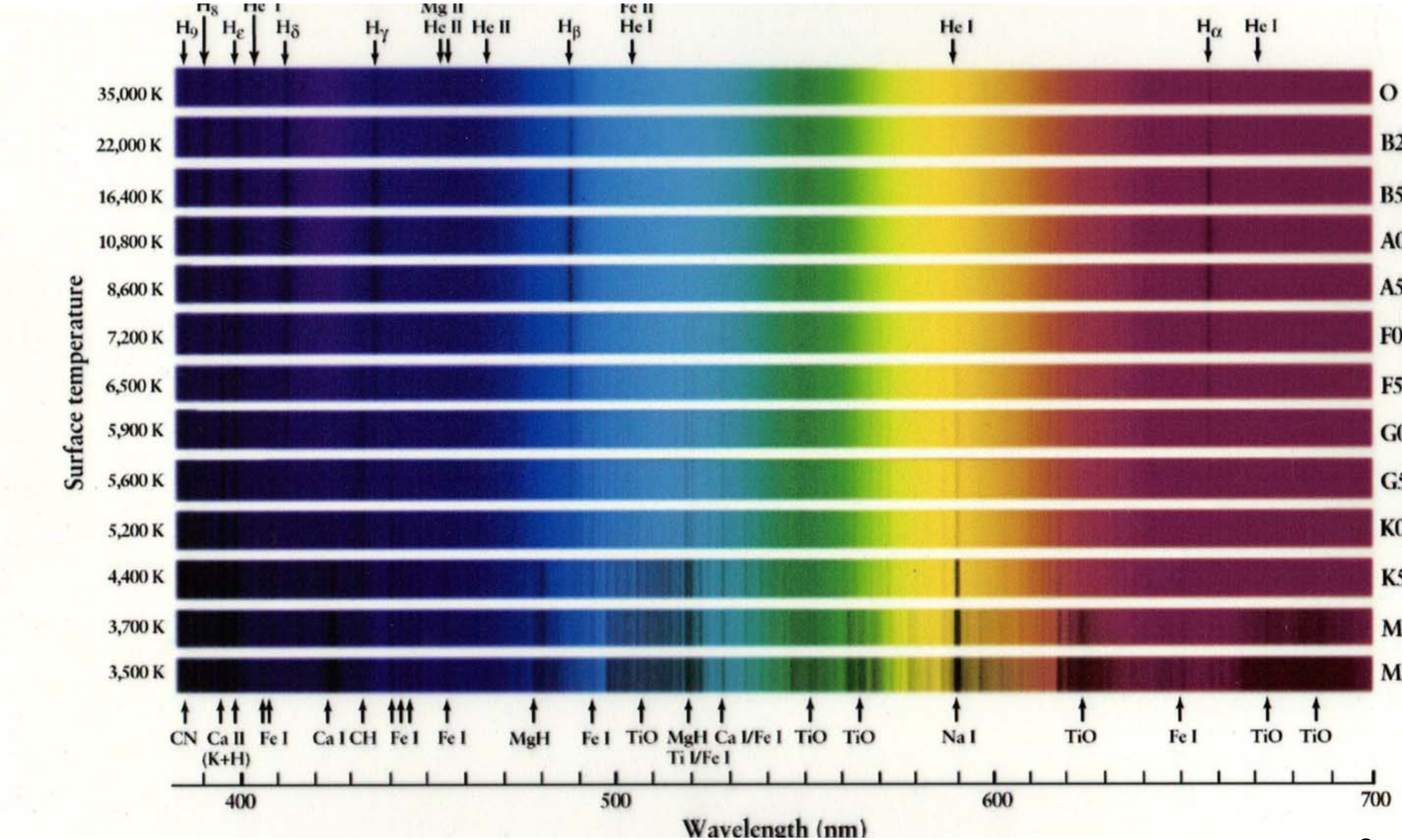


TABLE 8.1 Harvard Spectral Classification.

Temperature	Spectral Type	Characteristics	Notes
20-40,000K	O	Hottest blue-white stars with few lines Strong He II absorption (sometimes emission) lines. He I absorption lines becoming stronger.	Multiply ionized atoms e.g. CIII, NIII, Si V
15,000K	B	Hot blue-white He I absorption lines strongest at B2. H I (Balmer) absorption lines becoming stronger.	B9 – no HeI (~4030Å) Ca II (H & K) weakening
9,000K	A	White Balmer absorption lines strongest at A0, becoming weaker later. Ca II absorption lines becoming stronger.	
7,000K	F	Yellow-white Ca II lines continue to strengthen as Balmer lines continue to weaken. Neutral metal absorption lines (Fe I, Cr I).	still some TiO
5,500K	G	Yellow Solar-type spectra. Ca II lines continue becoming stronger. Fe I, other neutral metal lines becoming stronger.	Ca II lines strongest at GO
4,000K	K	Cool orange Ca II H and K lines strongest at K0, becoming weaker later. Spectra dominated by metal absorption lines.	K5 - TiO bands
3,000K	M	Cool red Spectra dominated by molecular absorption bands, especially titanium oxide (TiO) and vanadium oxide (VO). Neutral metal absorption lines remain strong.	
2500-1300K	L	Very cool, dark red Stronger in infrared than visible. Strong molecular absorption bands of metal hydrides (CrH, FeH), water (H_2O), carbon monoxide (CO), and alkali metals (Na, K, Rb, Cs). TiO and VO are weakening.	
<1300K	T	Coolest, Infrared Strong methane (CH_4) bands but weakening CO bands.	

S and C spectral types for evolved giant stars are discussed on page 466.

2MASS
SDSS

Critical lines to "follow"
Balmer series, He I, FeI, Ca II (H & K)

Note: In Table, terms "weaker" "stronger" can refer to either earlier or later spectral type.

Balmer lines increase in strength
λ's 6563 Å
4861 Å
4340 Å
4102 Å

Balmer jump

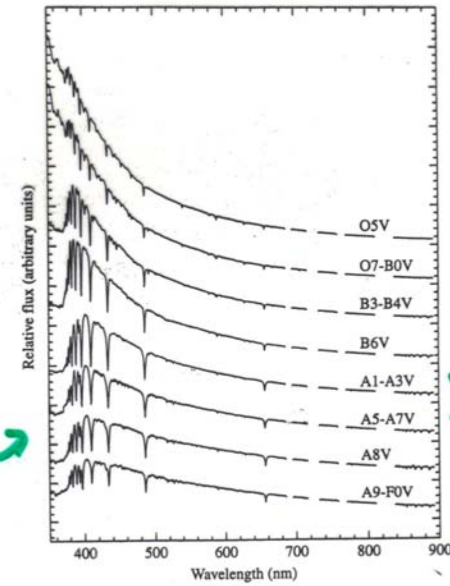


FIGURE 8.4 Digitized spectra of main sequence classes O5–F0 displayed in terms of relati
as a function of wavelength. Modern spectra obtained by digital detectors (as opposed to photog
plates) are generally displayed graphically. (Data from Silva and Cornell, *Ap. J. Suppl.*, *81*, 865,

Temp decreases for later spectral types

metals, molecules increase

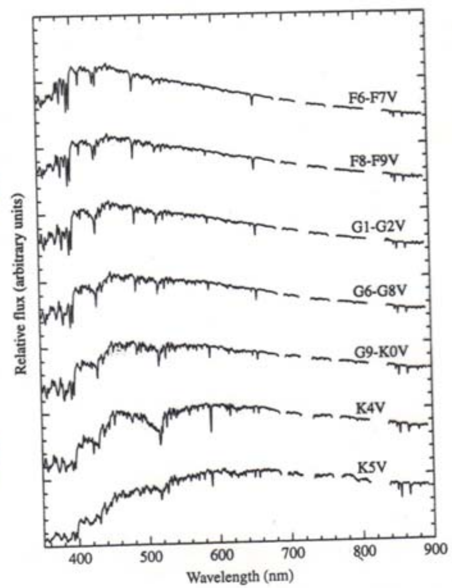


FIGURE 8.5 Digitized spectra of main sequence classes F6–K5 displayed in terms of relative
as a function of wavelength. (Data from Silva and Cornell, *Ap. J. Suppl.*, *81*, 865, 1992.)

Needed: a physical basis for spectral classification

For a particular element:

- What determines the relative numbers of atoms in each excitation state?
- What determines the relative numbers of atoms in each ionization state?

Statistical mechanics: an ensemble of gas particles

- for gas as a whole, T, P, ρ defined (temperature, pressure, density)
- behavior of individual particles, m, v, varies (mass, velocity)
- e.g. collisions $\rightarrow$ change in speed, energy

In thermal equilibrium, the fraction of particles having a specified range of velocities is described by the Maxwell-Boltzmann Distribution Function

number of particles with velocities between v and v+dv is:

$$n_v dv = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{\frac{-mv^2}{2kT}} 4\pi v^2 dv$$

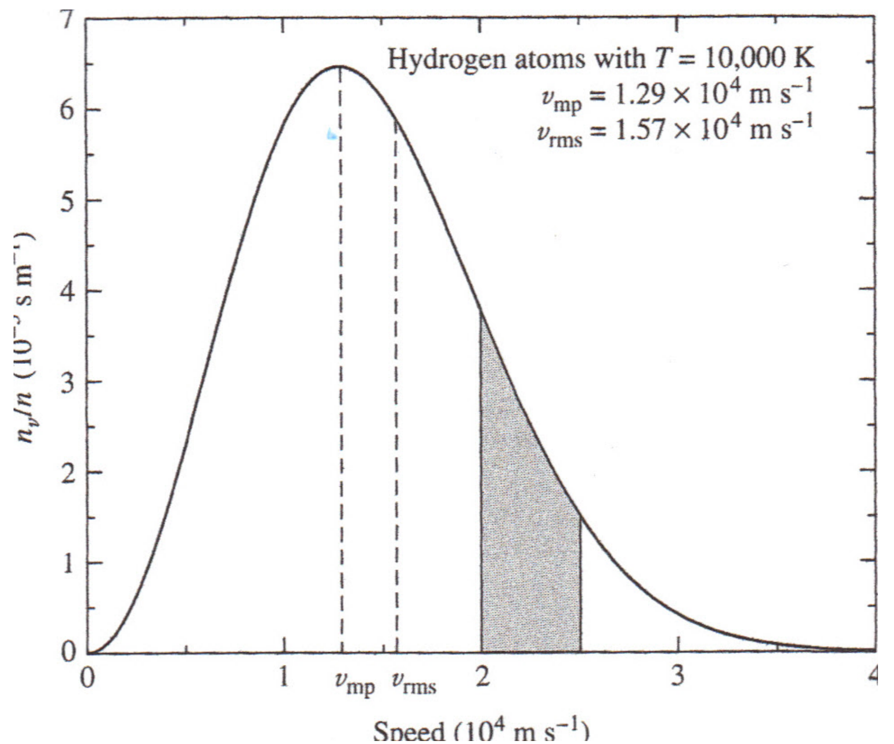
$$n_v dv = n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{\frac{-mv^2}{2kT}} 4\pi v^2 dv$$

n = number of particles/unit volume (number density),
n_v/n → fraction between v and +dv, k = Boltzmann's constant, T in °K

exponent = KE/thermal energy
= $\frac{1}{2}mv^2/kT$
most particles have thermal energy
(difficult to vary much from that)
∴ most probable speed for particles $v_{mp} = (2kT/m)^{1/2}$

including high speed tail of Maxwellian distribution → $v_{rms} = \sqrt{\frac{3kT}{m}}$

fraction of particles under curve between two speeds = fraction of gas particles with that range of speeds



Maxwell-Boltzmann distribution function n_v/n for H atoms at 10,000K

Example* here→ 12.5% of H atoms have speeds between 2 and 2.5 x 10^6 cm/sec

Relative numbers of atoms in each excitation state

As atoms collide, energy gained and lost

Maxwell-Boltzmann function governs distribution of speeds

Also a distribution of excitation states, with higher energy states less likely to be occupied

Suppose we have 2 energy states E_a and E_b, described by sets of quantum numbers s_a and s_b

Let $P(s_a)$ and $P(s_b)$ be probabilities that system is in state E_a or E_b

- $\therefore$

$$\frac{P(s_b)}{P(s_a)} = \frac{e^{-E_b/kT}}{e^{-E_a/kT}}$$

T is same for each state and $e^{-E/kT}$ is the Boltzmann factor

$$\frac{P(s_b)}{P(s_a)} = e^{-(E_b - E_a)/kT}$$

We have $P(s_b)/P(s_a) = e^{-(E_b-E_a)/kT}$

For $E_b > E_a$, as $T \to 0$, $E_b-E_a/kT \to -\infty$, $P(s_b)/P(s_a) \to 0$

- eventually no energy to raise atom to next excited state

and as $T \to \infty$, $E_b-E_a/kT \to 0$, $P(s_b)/P(s_a) \to 1$

- lots of energy; all states equally available

Energy levels can be degenerate i.e. more than one quantum state with same energy - could have $E_a = E_b$

But $s_a \neq s_b$ (Table 8.2 C & O)

In order to include *all* states (i.e. states with same E), let g_a = number of states with energy E_a, g_b with E_b etc

Statistical weight, g_n = number of states with energy E_n

$\therefore P(s_b)/P(s_a) = g_b e^{-E_b/kT}/g_a e^{-E_a/kT} = g_b/g_a e^{-(E_b-E_a)/kT}$

= ratio of probability that system is in any of g_b degenerate states to probability that it is in any of g_a degen. states

Statistical weights for H atom

Ground States s_1				Energy E_1
n	ℓ	m_ℓ	m_s	(eV)
1	0	0	+1/2	−13.6
1	0	0	−1/2	−13.6

First Excited States s_2				Energy E_2
n	ℓ	m_ℓ	m_s	(eV)
2	0	0	+1/2	−3.40
2	0	0	−1/2	−3.40
2	1	1	+1/2	−3.40
2	1	1	−1/2	−3.40
2	1	0	+1/2	−3.40
2	1	0	−1/2	−3.40
2	1	−1	+1/2	−3.40
2	1	−1	−1/2	−3.40

Statistical weight g_n= number of states with energy E_n

For H atom, g_1 = 2 and g_2 =8 for ground state and first excited state respectively

for H atom, degeneracy of energy levels level $g_n = 2n^2$ (C&O Chap 5)

Many atoms in stellar atmospheres

∴ ratio of probabilities ≡ ratio of number of atoms

$$P(s_b)/P(s_a) = g_b/g_a e^{-(E_b - E_a)/kT} = N_b/N_a$$

∴ relative numbers of atoms in two states of excitation from Boltzmann equation

$$N_b/N_a = g_b e^{-E_b/kT} / g_a e^{-E_a/kT} = g_b/g_a e^{-(E_b - E_a)/kT}$$

C&O problem 8.1.3: Gas has equal numbers of atoms in n=1 (ground) and n=2 (first excited) states at $T \approx 8.5 \times 10^4$ K

BUT observations → Balmer lines (upward transitions from n=2 level) most intense at T ~ 9250K

Why does Balmer line intensity decrease above 9250K?

PLOT CONFIRMS HIGH TEMPS NEEDED FOR SIGNIFICANT N_2

HYDROGEN

N_2/N_1+N_2

≡ RELATIVE OCCUPANCY OF GROUND AND FIRST EXCITED STATES

$N_1 \equiv N_a \quad N_2 \equiv N_b$

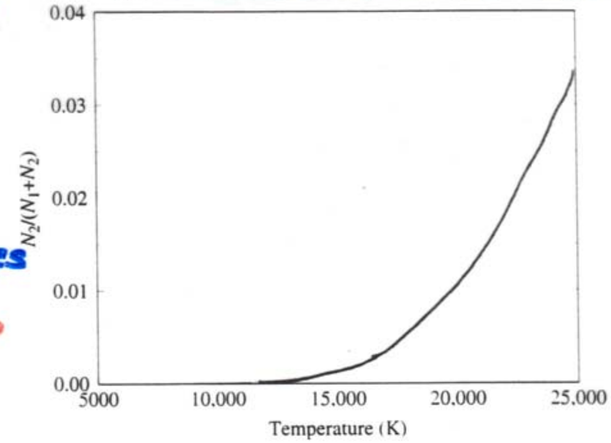


Figure 8.5 $N_2/(N_1+N_2)$ from the Boltzmann equation.

$N_2/N_1 = g_2/g_1 \, e^{-(E_2-E_1)/kT}$

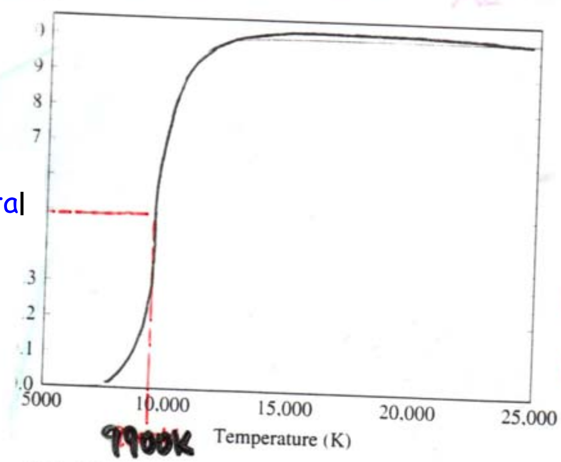
BUT

BALMER LINE STRENGH DEPENDS ON N_2/N_{TOTAL}

= FRACTION OF ALL H ATOMS IN FIRST EXCITED STATE

→ EFFECT OF ATOMS IN DIFFERENT IONIZATION STATES

N_{II}/N_{total}



ALMOST ALL HYDROGEN IONIZED ABOVE 1[redacted],000K

(50% at 9600K, 5% at 8300K)

i.e. in very small temperature interval

e 8.6 N_{II}/N_{total} for hydrogen from the Saha equation. with $P_e = 20\,N\,m^{-2}$

Number of atoms in different ionization states

χ_i = ionization energy required to remove electron from an atom in ground state ≡ionization state i to i+1

- For hydrogen χ_i = 13.6ev (HI to HII)

Define partition function Z = weighted sum of ways atom can distribute electrons with same energy

- more energetic levels have less weight in Boltzmann factor

If E_j energy of jth level, g_j degeneracy of level,

$$Z = \sum_{j=1}^{\infty} g_j e^{-\left(E_j - E_i\right)/kT}$$

Z_i , Z_{i+1} partition functions for atom in initial and final states

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$$

Saha equation

n_e, m_e number density and mass of free electrons; h = Planck constant

Saha Equation

$$\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

recall Boltzmann equation

$N_b/N_a = g_b e^{-E_b/kT}/g_a e^{-E_a/kT} = g_b/g_a e^{-(E_b-E_a)/kT}$

compare:

- $-(E_b-E_a) \rightarrow -\chi_i$; $g_b/g_a \rightarrow 2Z_{i+1}/n_e Z_i$
- n_e - free electrons have an effect
  factor of 2 due to 2 possible spins for free electron; as n_e increases, N_{i+1} decreases – more electrons enable more recombinations of ions
- additional term $\equiv$ number density of electrons with quantum energy ($h\nu$) $\approx$ thermal energy (kT)

Replace n_e with P_e (= $n_e kT$), electron pressure in Saha:

$$\frac{N_{i+1}}{N_i} = \frac{2kTZ_{i+1}}{P_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

Electron pressure very relevant in stellar atmospheres

The whole shebang: combining Saha and Boltzmann

Assume stellar atmosphere is pure H, and P_e = 20 N/m^2 (constant)

Saha equation* → N_{II}/N_{total} in terms of N_{II}/N_I

Plot (see page 11 here) shows H ionization levels:

0% at 5000k, 5% at 8300K, 95% at 11,300K

Ionization occurs within very limited range of T – barely 3000K

Now $N_2/N_{total} = N_2 \times (N_1 + N_2)^{-1} \times N_I/N_{total}$ (can assume NI = N1 + N2)

$$= N_2/N_1 (1 + N_2/N_1)^{-1} \times (1 + N_{II}/N_I)^{-1}$$

Largest fraction of atoms in n=2 level at T = 9900K

Decrease in fraction after 10,000K due to increasing ionization

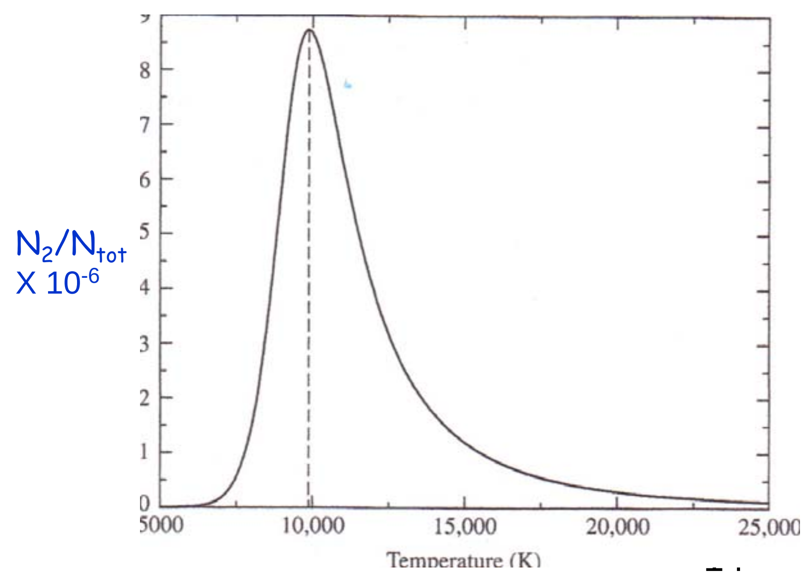
Saha-Boltzmann valid for other elements

Gas must be in thermal equilibrium for Maxwell Boltzmann distribution to apply

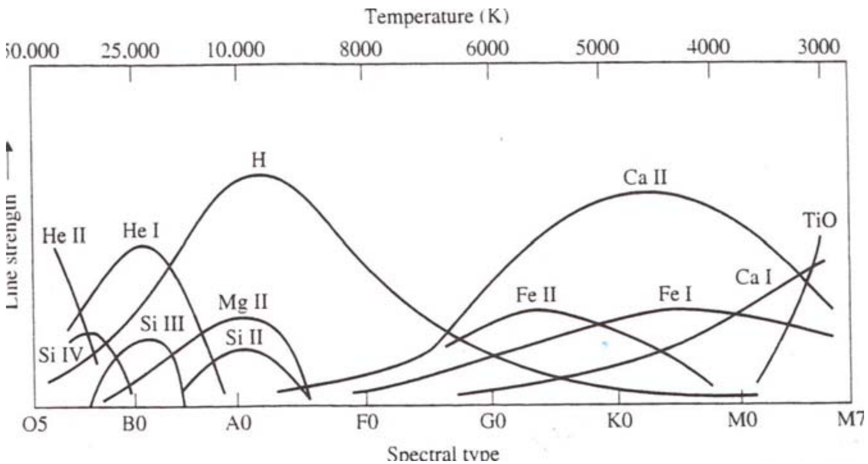
e.g. helium: He/H =1/10 by number

HeII → more free electrons, more HII recombinations

Example 8.1.5 C&O Ca H & K lines



- Most stars have similar relative abundances of elements as Sun
- Dominated by H, He, and then "metals"
- Spectral line intensities strongly dependent on T



Composition of Stars 1925
Cecilia Payne-Gaposhkin
Relative abundances of 18 elements

TABLE 9.2 The Most Abundant Elements in the Solar Photosphere. The relative abundance of an element is given by $\log_{10}(N_{el}/N_H) + 12$. (Data from Grevesse and Sauval, *Space Science Reviews*, *85*, 161, 1998.)

Element	Atomic Number	Log Relative Abundance
Hydrogen	1	12.00
Helium	2	10.93 ± 0.004
Oxygen	8	8.83 ± 0.06
Carbon	6	8.52 ± 0.06
Neon	10	8.08 ± 0.06
Nitrogen	7	7.92 ± 0.06
Magnesium	12	7.58 ± 0.05
Silicon	14	7.55 ± 0.05
Iron	26	7.50 ± 0.05
Sulfur	16	7.33 ± 0.11
Aluminum	13	6.47 ± 0.07
Argon	18	6.40 ± 0.06
Calcium	20	6.36 ± 0.02
Sodium	11	6.33 ± 0.03
Nickel	28	6.25 ± 0.04

LATER: fractions by weight — H: X 0.70; He: Y 0.28; Metals: Z 0.02

[32]Details of the construction of a model star will be deferred to Chapter 10.

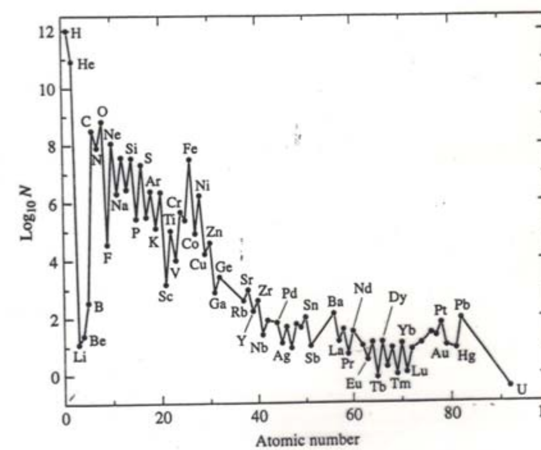


FIGURE 15.16 The relative abundances of elements in the Sun's photosphere. All abundances are normalized relative to 10^{12} hydrogen atoms. (Data from Grevesse and Sauval, *Space Sci. Rev.*, *85*, 161, 1998.)