AY 20 Fall 2010

Electromagnetic Radiation and the Properties of Stars

Reading: Carroll & Ostlie, Chapter 3

Physical constants

Symbol	Description	SI		cgs	
		Value	Unit	Value	Unit
c	Speed of light	2.9979 (8)	m s^{-1}	2.9979 (10)	cm^{-1} s^{-1}
h	Planck's constant	6.6261(−34)	J s	6.6261(−27)	erg s
k	Boltzmann's constant	1.3807(−23)	J/K	1.3807(−16)	erg/K
σ_{SB}	Stefan–Boltzmann constant	5.6704 (−8)	W m^{-2} K^{-4}	5.6704 (−5)	erg s^{-1} cm^{-2} K^{-4}
G	Gravitational constant	6.674 (−11)	N m^{-2} kg^{-2}	6.674 (−8)	dyn cm^{-2} g^{-2}
N_A	Avogadro's constant	6.0221 (23)	mol^{-1}	6.0221 (23)	mol^{-1}
m_e	Electron rest mass	9.1094(−31)	kg	9.1094(−28)	g
m_p	Proton rest mass	1.6726(−27)	kg	1.6726(−24)	g
m_u	Atomic mass unit	1.6605(−27)	kg	1.6605(−24)	g
e	Electron charge	1.602 (−19)	C	4.803 (−10)	esu
α	Fine-structure constant	7.2974 (−3)		7.2974 (−3)	

Values $a \times 10^b$ are given as a (b).

Astronomical constants

Symbol	Description	SI		cgs	
		Value	Unit	Value	Unit
AU	Astronomical unit	1.496 (11)	m	1.496 (13)	cm
ly	Light year	9.463 (15)	m	9.463 (17)	cm
pc	Parsec	3.086 (16)	m	3.086 (18)	cm
pc^2	Square parsec	9.5234 (32)	m^2	9.5234 (36)	cm^2
kpc^2	Square kiloparsec	9.5234 (38)	m^2	9.5234 (42)	cm^2
$L_\odot$	Solar luminosity	3.85 (26)	J s^{-1}	3.85 (33)	erg s^{-1}
$M_\odot$	Solar mass	1.989 (30)	kg	1.989 (33)	g
$R_\odot$	Solar radius	6.96 (8)	m	6.96 (10)	cm
$T_\odot$	Solar effective temperature	5.78 (3)	K	5.78 (3)	K
Jy	Jansky	1.00 (−26)	W m^{-2} H z^{-1}	1.00 (−23)	erg s^{-1} cm^{-2} Hz^{-1}

Values $a \times 10^b$ are given as a (b).

Stellar Properties

From measurements of radiation at visible wavelengths:

stellar motions √

brightness

distance

"temperature"

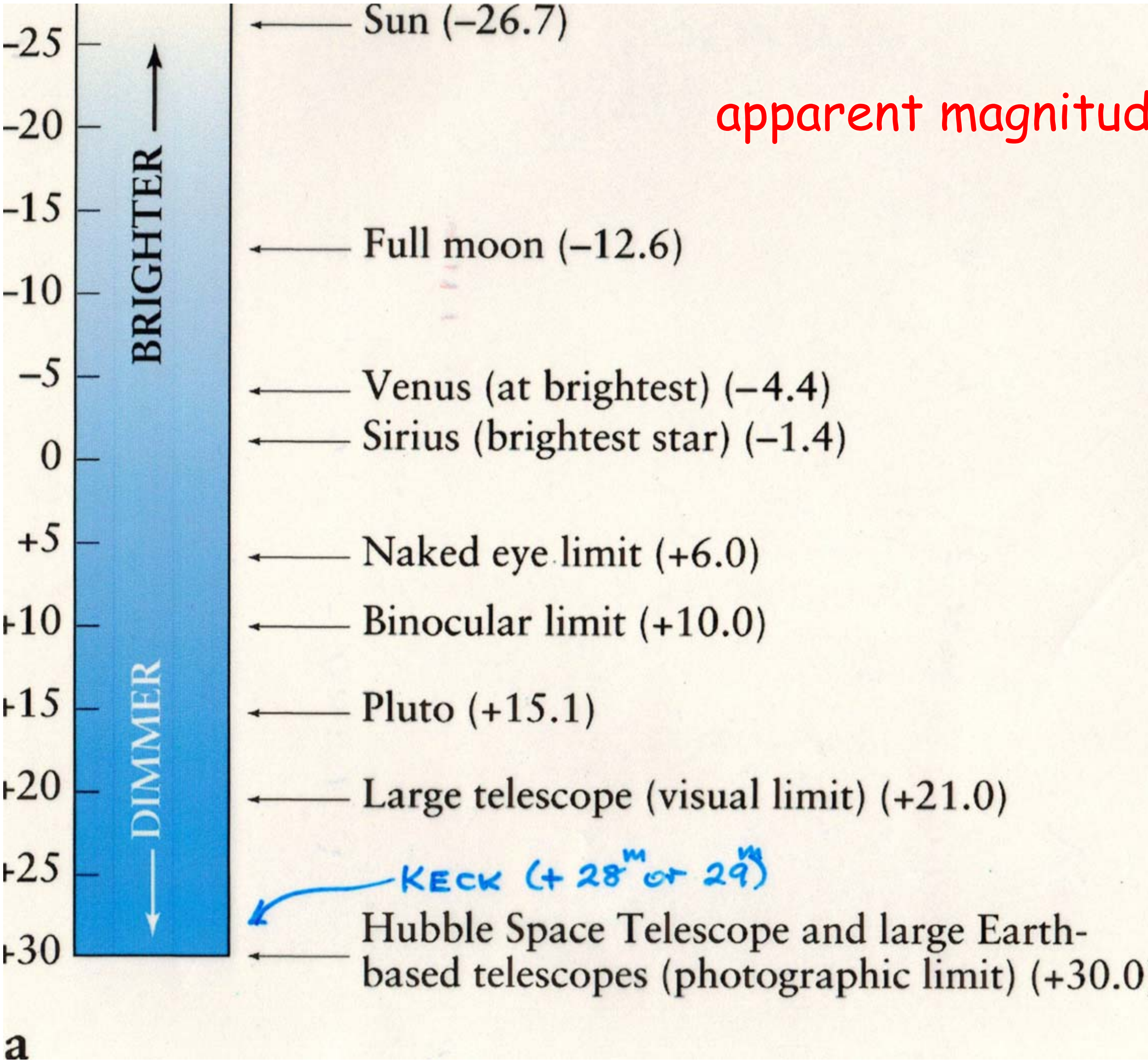
brightness $B = L/4\pi d^2$, L = luminosity (ergs/sec), d= distance to star

B measured in magnitudes

$$m_1 - m_2 = 2.5 \log b_2 / b_1$$

each factor 100 in brightness = 5^m

apparent magnitudes



Stellar Properties

From measurements of radiation at visible wavelengths: stellar

motions √

brightness

distance

"temperature"

Brightness, B measured in magnitudes

$$m_1 - m_2 = 2.5 \log b_2 / b_1$$

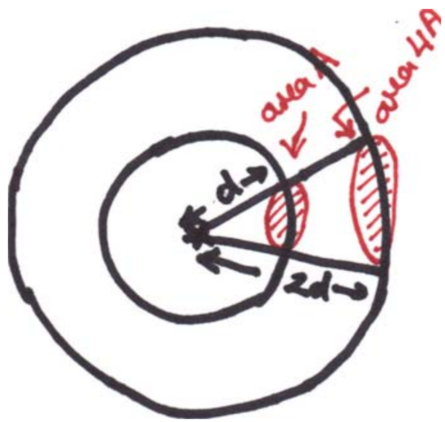
each factor 100 in brightness = 5^m

Need a brightness scale independent of distance

Define Absolute Magnitude, M

M = magnitude of a star at a distance of 10 pc

Flux and Brightness



photometers measure radiant flux, F_λ
= brightness

F = amount of energy crossing unit area in unit time ($ergs/cm^2/sec$)

Luminosity = energy/sec from star
= $4\pi d^2 F$ (d= distance from star)

$\therefore F = L/4\pi d^2$

i.e. $F \propto 1/d^2$ (inverse square law for light)

Note: L is an intrinsic property of star

Solar flux at earth, $F_\odot = 4\times 10^{33}/4\pi(1.5\times 10^{13})^2 \sim 1.4\times 10^6$
= solar constant (1.36×10^6 $ergs/cm^2/sec$)

Absolute magnitude → Distance Modulus

$\underline{M}$ = <u>magnitude of a star at a distance of 10 pc</u>

$m_1 - m_2 = 2.5\log b_2/b_1 = 2.5\log F_2/F_1 = -2.5\log F_1/F_2$

For a star of luminosity L seen at distances d_1 and d_2,

$$F_1 = L/4\pi d_1^2 \text{ and } F_2 = L/4\pi d_2^2$$

$$\therefore m_1 - m_2 = -2.5\log F_1/F_2 = -2.5\log(d_2/d_1)^2$$

Let m be observed magnitude of the star at distance d pc

$$\therefore m - M = -2.5\log(10/d)^2$$

$$\therefore m - M = 5\log d - 5$$

$$\text{or, } d = 10^{[(m-M+5)/5]} \text{ pc}$$

m-M = distance modulus

L and M are intrinsic to star

F and m affected by distance

Luminosity and Absolute Magnitude Relation

For the Sun: $m = -26.8$ and $d = 1$ AU

$$m - M = 5\log d - 5$$

$\therefore M_\odot = -26.8 + 5 - 5\log(1.5 \times 10^{13})/(3.1 \times 10^{18})$

$= -26.8 + 5 - 5\log(5 \times 10^{-6}) = -21.8 + 5 - 3.5 + 30 = 4.7$ (4.74)

$\therefore$ for Sun, distance modulus $= 4.7 + 26.8 = 31.5$

$$F = L/4\pi d^2$$

$\therefore F_1/F_2 = L_1/L_2$, for 2 stars at same distance

$$\therefore m_1 - m_2 = -2.5\log F_1/F_2 = -2.5\log L_1/L_2$$

$$\therefore M_* - M_\odot = -2.5\log L_*/L_\odot ,$$

with $M_\odot = 4.7$ and $L_\odot = 3.9 \times 10^{33}$ ergs

Spectroscopic Parallax:

distance modulus +stellar spectrum $\rightarrow$ distance to star

Spectral type (later) $\rightarrow M_\lambda$

Observation $\rightarrow m_\lambda$

$$m_\lambda - M_\lambda = 5\log d - 5$$

Bolometric Magnitude:

star (black body) emits radiation across wide range of λ

bolometric magnitude measured over all emitting wavelengths

m_{bol} and M_{bol}

Blackbody Radiation

Continuous emission spectrum from a blackbody

Peak emission wavelength, λ_{max}, varies with temperature T of B-B

λ_{max} decreases with increasing T

Wien's law (empirical)

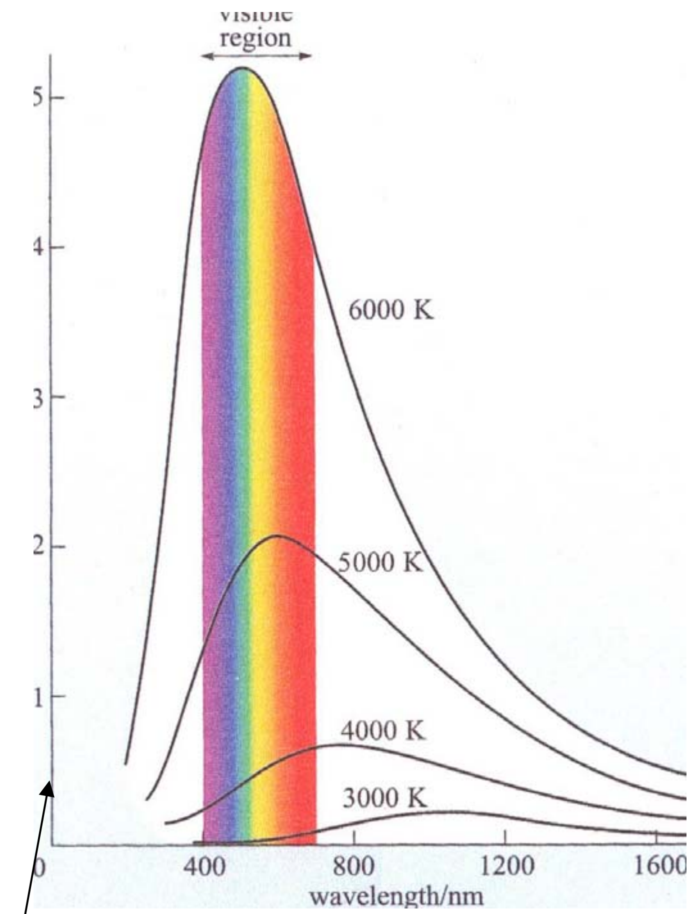
$\lambda_{max} = 0.0029/T$ (λ m; T °K)

OR $\lambda_{max}T = 0.29$ K (λ cm)

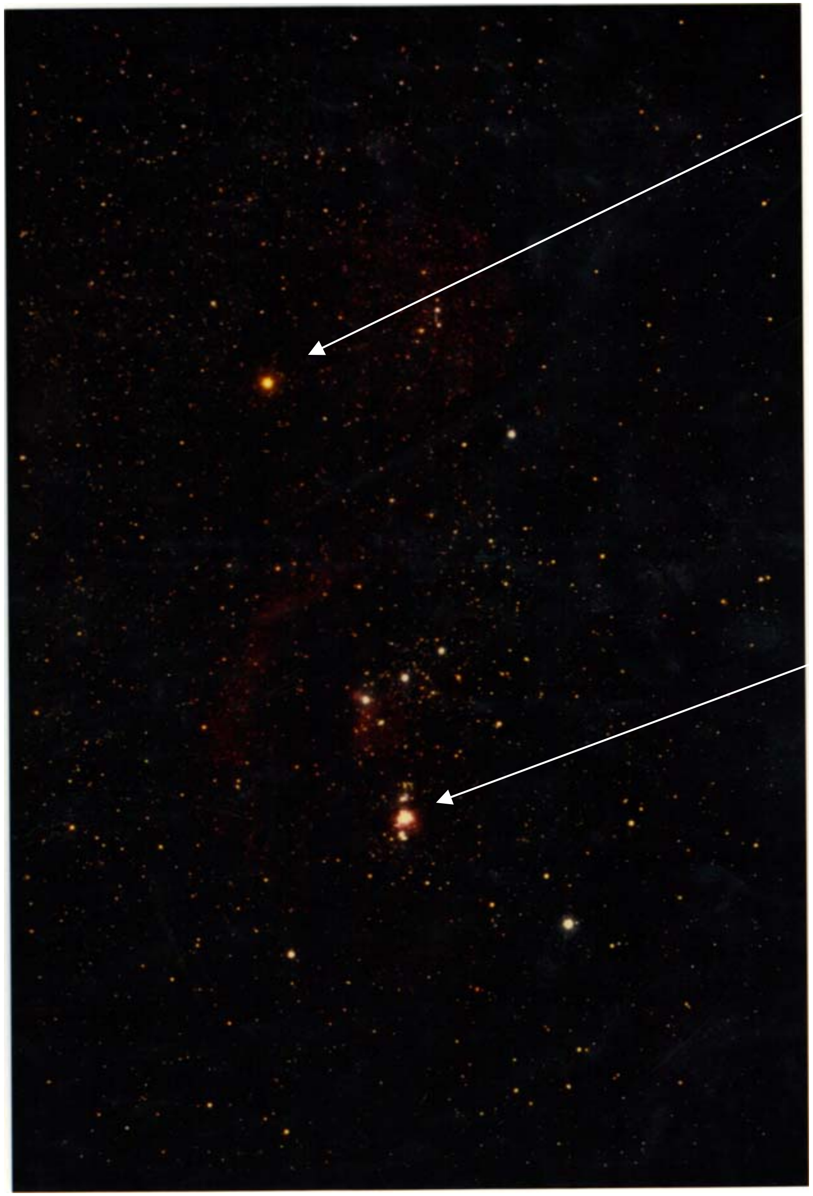
peak of curve → surface temp

cooler objects are redder

Spectra of blackbody sources



Relative flux density in arbitrary units



Betelgeuse (Alpha Orionis)

$T_{surface}$ = 3600 K

spectral type M

d ~ 140 pc

R ~ 660 $R_\odot$

$0.45 \times 10^4 L_\odot < L > 1.5 \times 10^4 L_\odot$

Rigel (Beta Orionis)

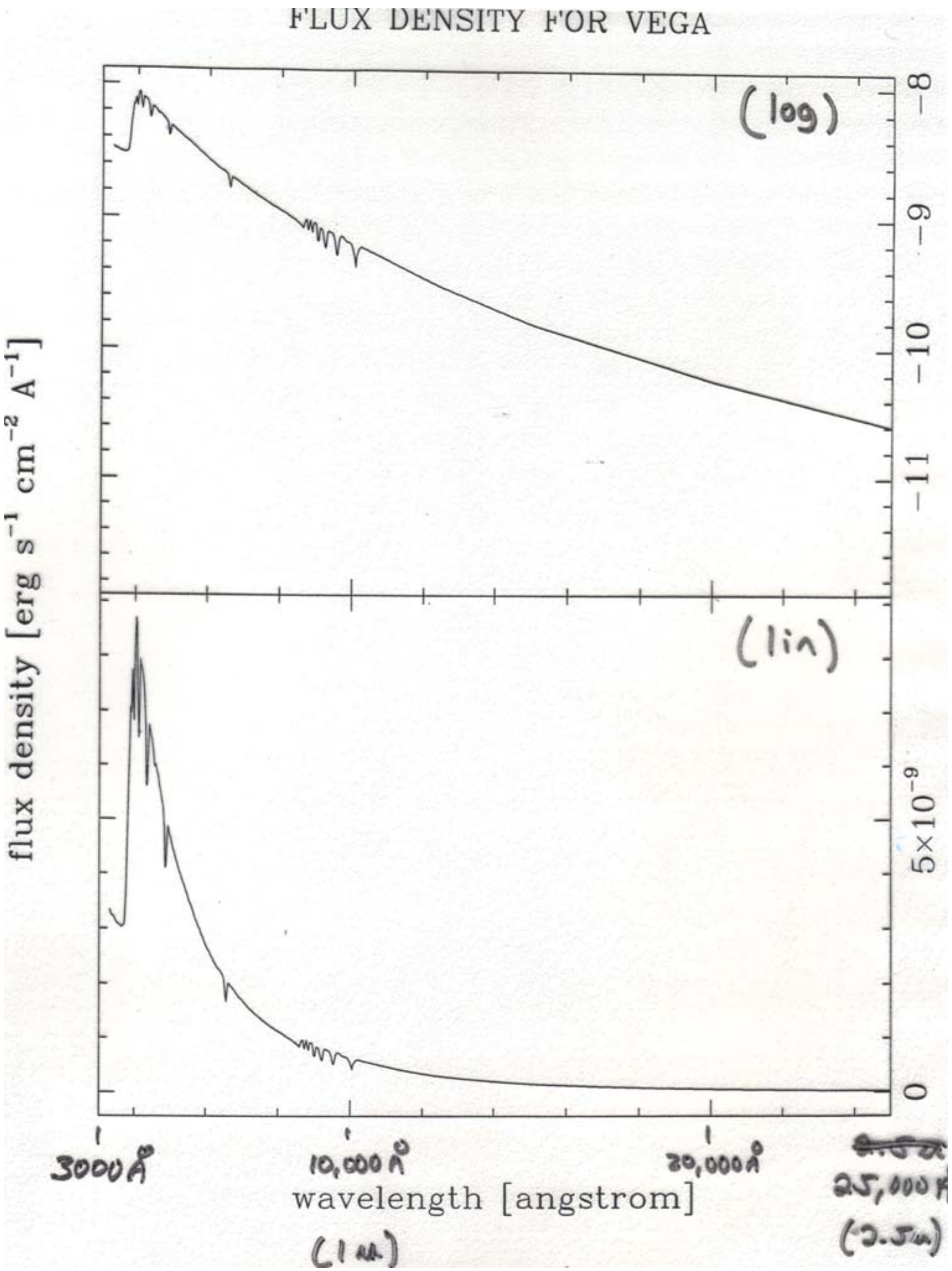
$T_{surface}$ = 11,000 K

spectral type B8

d = 260 pc

R ~ 60 $R_\odot$

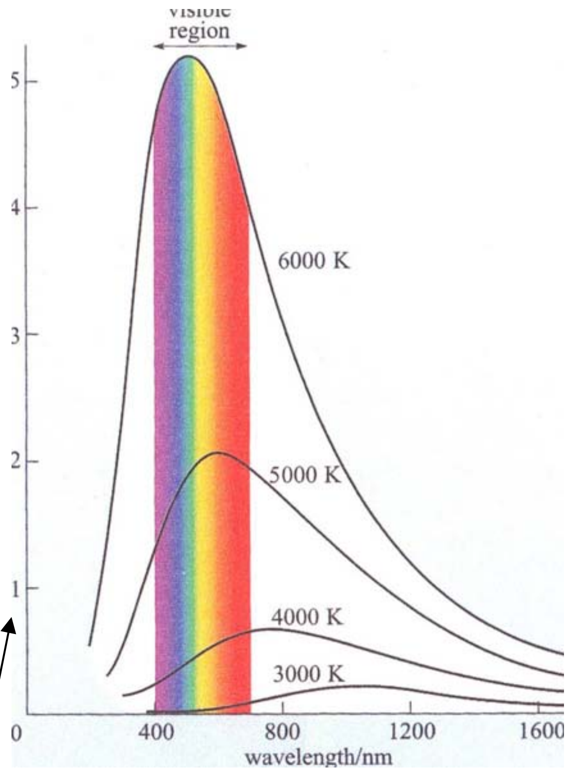
$L \sim 3.9 \times 10^4 L_\odot$



Vega (Alpha Lyrae)

T = **9500 K**
spectral type AO
d ~ 70 pc

Blackbody Radiation cont'd



Relative flux density in arbitrary units

- At all wavelengths, emission/sec increases with increasing T
- Stefan (empirically): $F = \sigma T^4$

$\therefore$ $L = A\sigma T^4$ (A = surface area of B-B)

σ= 5.67 x 10^{-5} ergs/sec/cm^2/K^4 (Boltzmann)

Stefan- Boltzmann equation:

$L = A\sigma T^4$

$\therefore$ $L= 4\pi R^2 \sigma T_e^4$ for a star of radius R

(T_e = effective temperature of stellar surface)

Page 6: $F \propto 1/d^2$ where d is distance from luminosity source

At surface of star, d = R

$\therefore$surface flux $F_{surface} = L/ 4\pi R^2$

$\therefore F_{surface} = \sigma T_e^4$

A Physical Basis for the B-B Radiation Curve?

Rayleigh: $B_\lambda(T) \approx 2ckT/\lambda^4$ k = Boltzmann's constant (PV = NkT)

OK in radio; ultraviolet catastrophe as $\lambda \to 0$

Rayleigh-Jeans law: $B_\lambda(T) \approx 2ckT/\lambda^4$ for long λ

Wien's law: $B_\lambda(T) \approx a\lambda^{-5}e^{-b/\lambda kT}$ for short λ

a, b, constants ; fit experimental data

Planck (1900): assumed standing wave of wavelength λ & frequency $\nu = c/\lambda$ could acquire only integral values of some minimum energy (quantum), $h\nu$ or hc/λ

h = Planck's constant =6.6×10^{-27} erg sec

$$B_\lambda(T) = 2h\frac{c^2}{\lambda^5}\left(e^{hc/\lambda kT} - 1\right)^{-1}$$

B measured in W m^{-2} m^{-1} $steradian^{-1}$ or ergs s^{-1} cm^{-2} cm^{-1} sr^{-1}

Deriving laws from Planck function

Can express Planck function in terms of frequency.

Use $B_\nu d\nu = - B_\lambda d\lambda$ and $\nu = c/\lambda$, $d\lambda/d\nu = -c/\nu^2$

$\therefore B_\nu = -B_\lambda(d\lambda/d\nu) = B_\lambda(c/\nu^{2)}$

$$B_\nu(T) = 2h\frac{\nu^3}{c^2}\left(e^{h\nu/kT} - 1\right)^{-1}$$

Integrating $B_\nu(T)$ with $x = h\nu/kT$ and $d\nu = (kT/h)dx$,

Total intensity, $B(T) = (2k^4T^4/c^2h^3).(\pi^4/15) = AT^4$

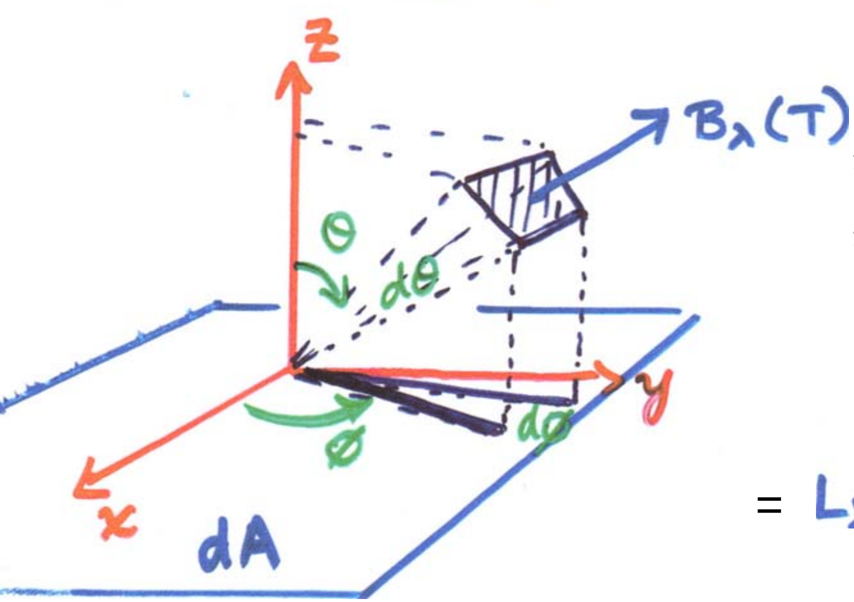
For isotropic radiation, $F = \pi B \therefore F = \sigma T^4$

$\equiv$Stefan Boltzmann Law

$\sigma = \pi A = 5.67 \times 10^{-5}$ ergs/sec/cm^2/K^{-4}

Can also derive Wein and Rayleigh-Jeans approximations

Planck Function relates star's observed properties (F, magnitude) to intrinsic (R, T)



e.g. model star, radius R, temp T

Each small patch of surface, dA, emits isotropically outwards

Energy emitted/sec between λ and $\lambda + d\lambda$ into solid angle $d\Omega$

$= B_\lambda(T) d\lambda dA \cos\theta d\Omega$

$= B_\lambda(T) d\lambda dA \cos\theta \sin\theta d\theta d\phi$

$\therefore$ Total energy/sec emitted between λ, $\lambda + d\lambda$

$$= L_\lambda d\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \int_{A} B_\lambda \, d\lambda \, dA \cos\theta \sin\theta \, d\theta \, d\phi$$

$= L_\lambda d\lambda$ = monochromatic luminosity

$= \pi 4\pi R^2 B_\lambda d\lambda = 4\pi^2 R^2 B_\lambda d\lambda$

angular integration

area of sphere

- $L_\lambda d\lambda = 4\pi^2 R^2 B_\lambda d\lambda$
- $\therefore L = 4\pi^2 R^2 \int B_\lambda(T) d\lambda$ over all wavelengths
- but $L = 4\pi R^2 \sigma T_e^4$ (Stefan-Boltzmann page 13)
- $\therefore \int B_\lambda(T) d\lambda = \sigma T_e^4/\pi$ (limits of integration 0 to ∞)

- Monochromatic flux = $F_\lambda d\lambda = L_\lambda / 4\pi^2 r^2 \, d\lambda$, r = distance to star

$$\therefore F_\lambda d\lambda = \frac{4\pi^2 R^2 B_\lambda d\lambda}{4\pi r^2} = \frac{2\pi hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}\left(\frac{R^2}{r^2}\right) d\lambda$$

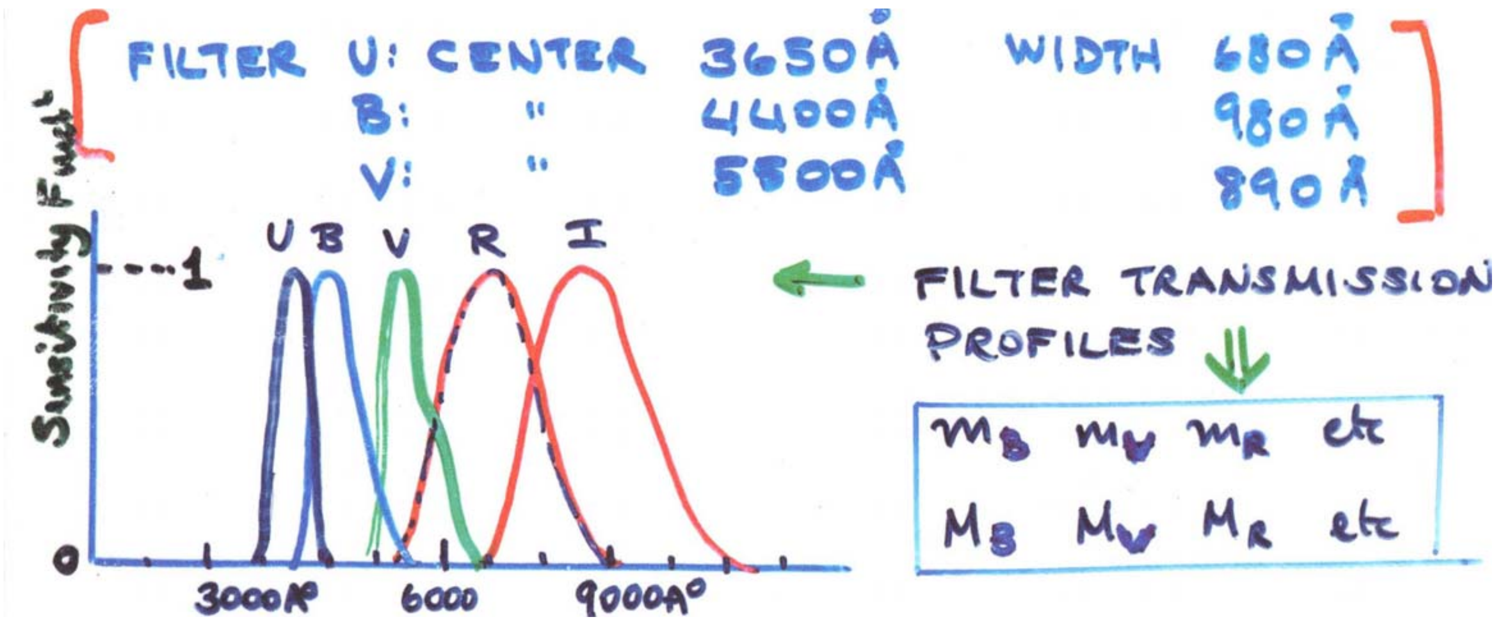
$\therefore F\lambda d\lambda$ is energy of starlight (in joules) with wavelength between λ and $\lambda + d\lambda$ arriving/sec at 1 sq meter of detector

In practice, flux measured over limited wavelength ranges using filters

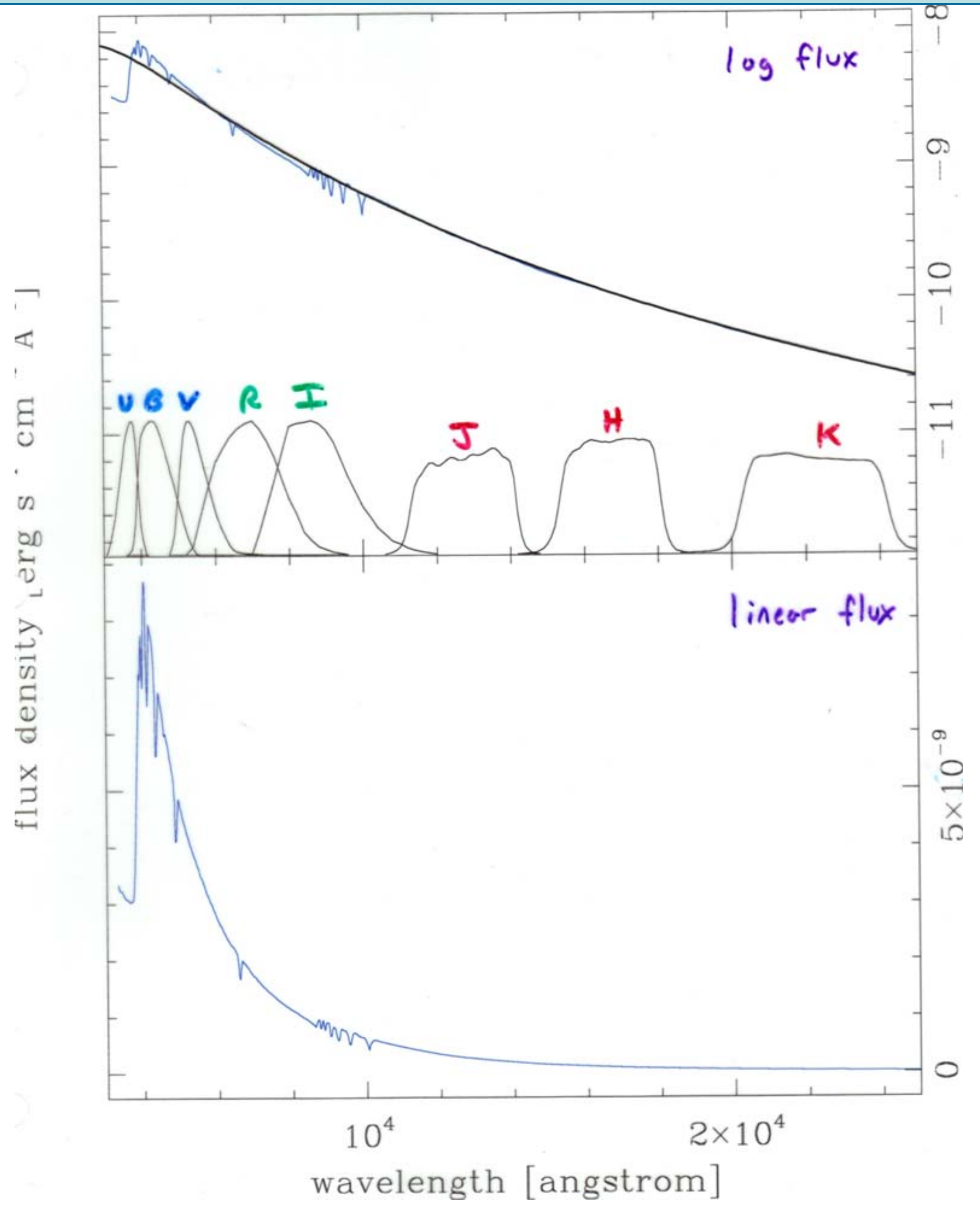
Define sensitivity function, S_λ = fraction of stellar flux detected at λ (affected by mirror reflectivity, bandwidth, detector response…)

$$\therefore F = \int F_\lambda S_\lambda d\lambda$$

Bandwidth: filters confine range of λ observed



Flux Density Measurements for Vega



Color Index

- Define Color Index:
  - U-B = $M_U - M_B$ or B-V = $M_B - M_V$ etc
    - Recall Wien Law: $\lambda_{max}T = 0.29$
  - i.e. λ_{max} measures stellar temperature
- Relative values of M_U M_B $M_{V...}$ also indicate temperature
  - e.g. *smaller* B-V = bluer = hotter