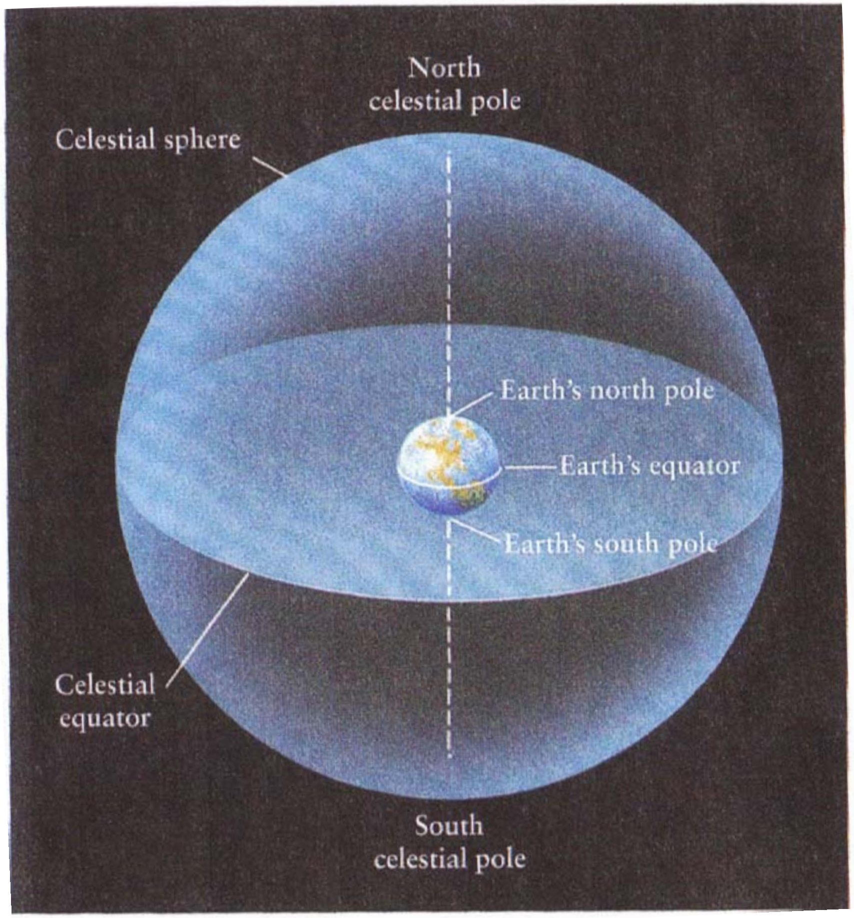
AY 20 Fall 2010

Celestial Sphere concluded
Celestial Mechanics
Electromagnetic Radiation and the Properties of Stars

Reading: Carroll & Ostlie, Chapters [1],2,3

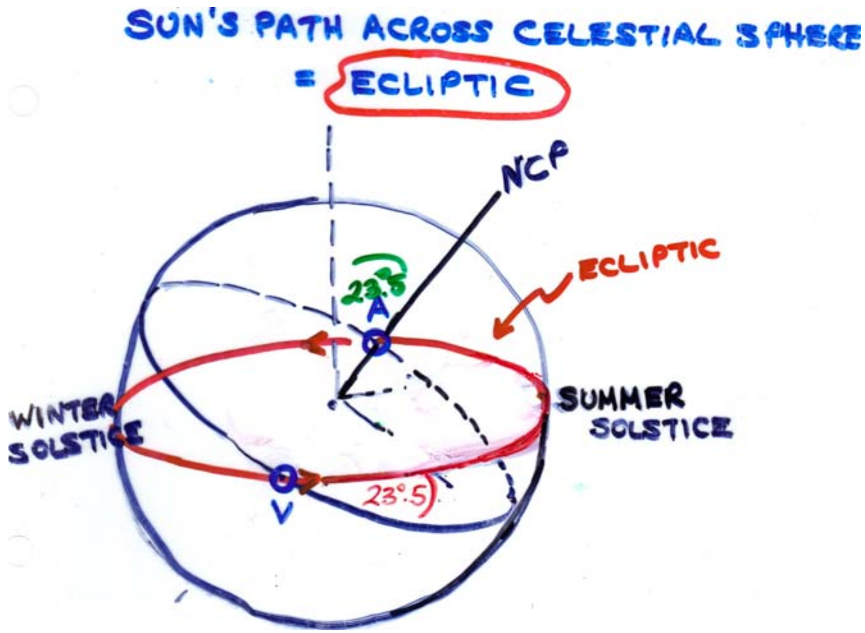
From last class:



celestial sphere

- Altitude-Azimuth coordinate system too "local"
- Equatorial coordinate system enables catalogs of star/galaxy positions for use anywhere
- Astronomical observations based on earth-centered reference frame have to take into account
  - Diurnal rotation
  - Annual motion around Sun
  - Wobble of rotation axis
  - Stars move relative to one another

Last class: Earth's axis tilted w.r.t. NCP → seasons



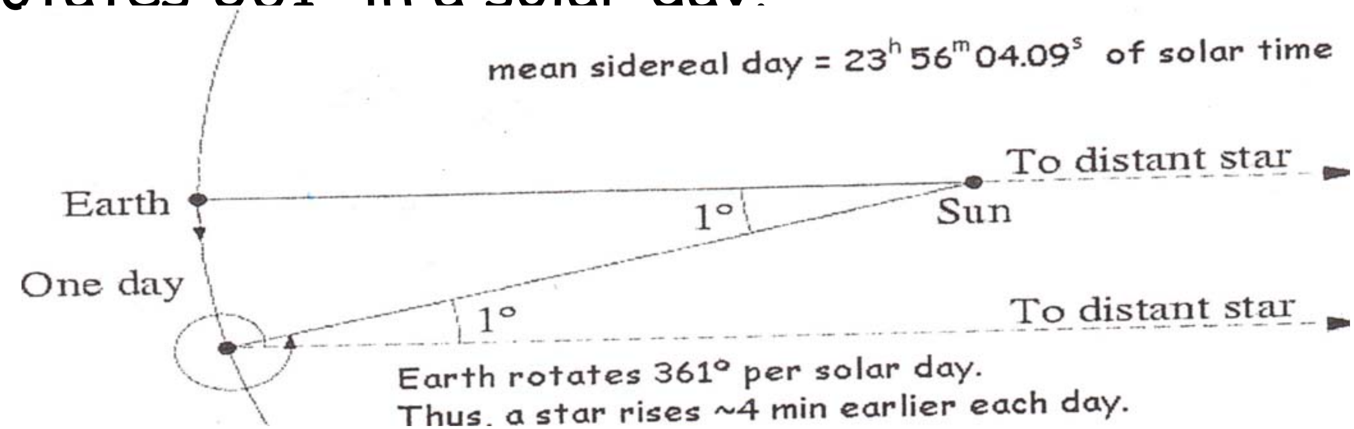
- During a year Sun moves north and south of celestial equator
- Crossing points of ecliptic at vernal and autumnal equinoxes
  - vernal (sun moving north); autumnal (sun moving south)
- Summer solstice: sun at its greatest northern declination
- Winter solstice: sun at its greatest southern declination
- Each hemisphere alternately points towards and away from Sun

Vernal equinox defines origin of right ascension.

Sun is on meridian at noon with RA = $00^h\ 00^m\ 00^s$

Effects of Rotation around Sun

- Sidereal day: time for individual stars to re-cross observer's local meridian
- Solar day = time for Sun to re-cross local meridian
- But Earth moves on its orbit by ~ 1° per day so rotates 361° in a solar day.



1° rotation = 4 minutes of time & 1 sidereal day = 23h 56m 4.09s

stars rise 4 minutes earlier each day

- Local Sidereal Time (LST) = time since ϒ (RA 00h 00m 00s) last passed the observer's meridian
- Equivalently: LST = hour angle (HA) of vernal equinox,

  HA = angle between star and observer's meridian measured in direction of star's motion around celestial sphere

Precession = wobble of Earth's rotation axis

- Slow wobble
  - due to non-spherical Earth (equatorial bulge) & gravitational pull of Sun, Moon
  - NCP, SCP change positions relative to stars over precession period (26,000 yrs); equinoxes also move

    North star = Polaris (~1° from NCP); in 12,000 years = Vega at >45°

    2000 years ago vernal equinox, Υ (Aries) ; currently in Pisces
- Reference Epoch of RA and dec must be specified

  $\rightarrow \alpha(2000)$, $\delta(2000)$ = position at noon GMT (Universal Time UT) on January 1, 2000

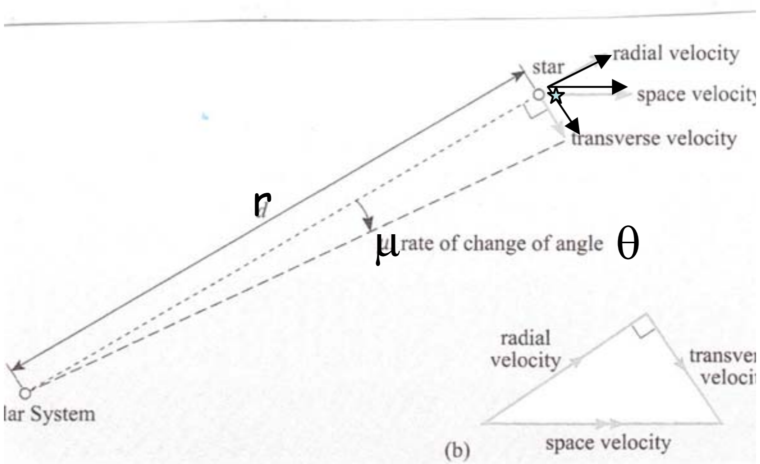
  Current positions from $\Delta\alpha = [m + n \sin\alpha \tan\delta]N$ $\Delta\delta = [n \cos\alpha]N$

  N=number of years since 2000

  m ~ 3.07″/yr and n ~ 20.04″/yr
- Expressed as J2000 when based on Julian calendar

  See example 1.3.1 Carroll and Ostlie

Motions of Stars



- v_r = radial velocity
  (velocity in line of sight)
- v_θ = transverse velocity
  (velocity along celestial sphere)
  = angular change in coords
  = proper motion, μ
  $\mu \equiv d\theta/dt$ (arcsecs/yr)

In time Δt, star moves transverse distance $\Delta d = v_\theta \Delta t$

angular change $\Delta\theta = \Delta d/r = v_\theta \Delta t/r$

$$\therefore \mu = d\theta/dt = v_\theta/r$$

Relation to equatorial coords

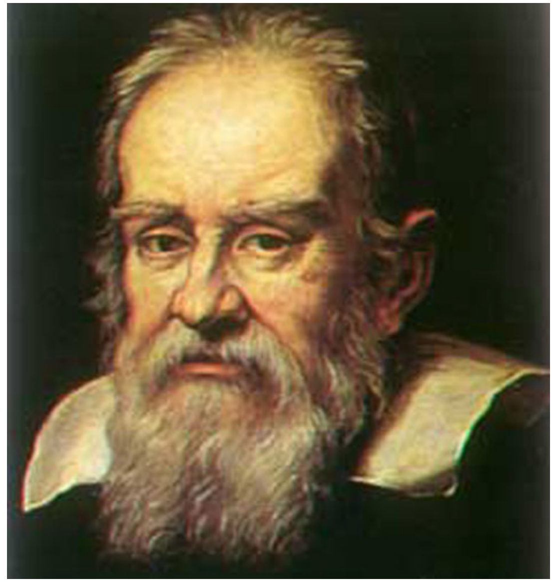
$$\Delta\alpha = \Delta\theta \sin\phi/\cos\delta,$$

$$\Delta\delta = \Delta\theta \cos\phi$$

where ϕ is direction of travel

A paradigm change – the birth of modern science

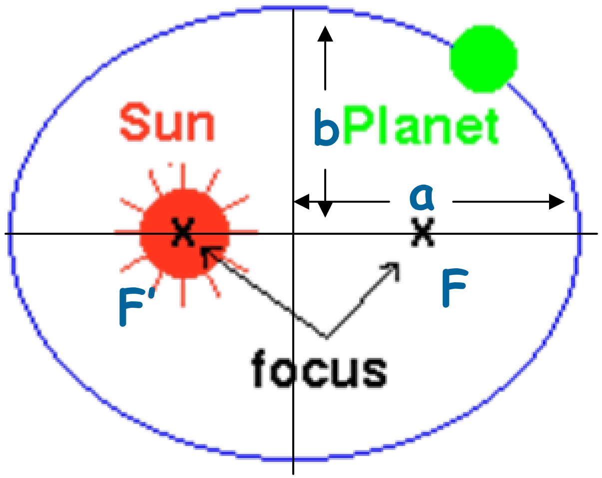
Johannes Kepler
1571-1630
Planets move in *elliptical* orbits
(Tycho Brahe's Observations)



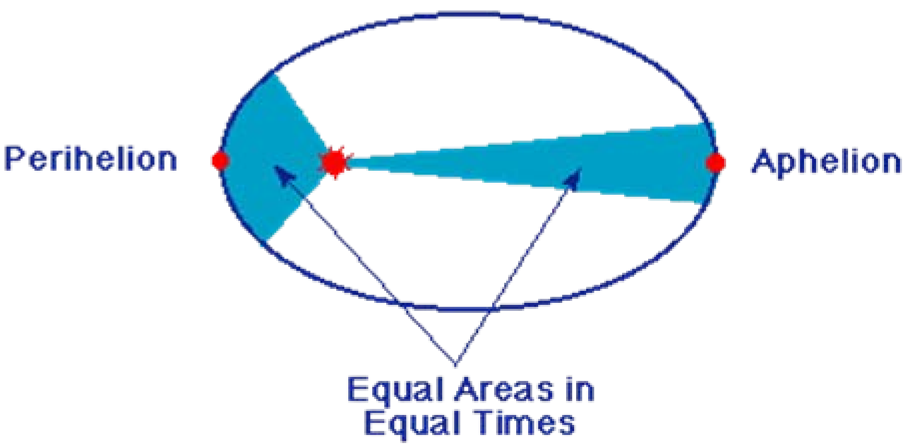
Galileo Galilei
1564-1642
Father of experimental physics
Experiment leads to fundamental principles

Isaac Newton
1642-1727
Mathematical principles enable explanation of observations/experiments

Kepler's Laws – empirical



a, b, semi-major and semi minor axes



1st and 2nd (1609)

1. The orbit of a planet is an ellipse with the Sun located at one focus

   eccentricity e= F-F'/2a

   For circle e =o

2. A line connecting a planet to the Sun sweeps out equal areas in equal times

   Planet orbital speed depends on position in orbit

3. The Harmonic Law (1620)

   The square of the period of revolution of a planet is proportional to the cube of the semi-major axis of its elliptical orbit

$$P^2 = a^3$$

P in years , a in Astronomical Units AU

1AU = Sun-Earth distance, 1.5×10^{11} m

Newton: laws of motion + law of gravitation provide theoretical underpinning for Kepler's laws

1. Law of Inertia: In the absence of an external force a particle will remain at rest or move in a straight line at constant velocity

   momentum **p** = m**v** is constant, unless there is an external force

2. The net force acting on a particle is proportional to the object's mass and and its resultant acceleration

   the rate of change of momentum of a particle is equal to the net force applied $\mathbf{F}_{net} = \Sigma^{n}_{i=1}\mathbf{F}_i = d\mathbf{p}/dt = md\mathbf{v}/dt = m\mathbf{a}$

3. For every action there is an equal and opposite reaction

   $F_{12} = F_{21}$

Newton's 3 Laws + Kepler's 3rd Law lead to Law of Gravitation

Universal Law of Gravitation

For 2 bodies, masses M and m, separated by r, the mutual gravitational force , F, is given by: $F = GMm/r^2$

G, gravitational constant = 6.673×10^{-8} dynes/cm^2/gm^2 (6.673 Newtons/m^2/kg^2)

Example 2.2.1 Carroll and Ostlie: force exerted on a point mass by a spherically symmetric mass also $F= GMm/r^2$ (all mass of larger body in effect concentrated at center)

Kepler's Laws can now be derived (C & O §2.3) $\rightarrow$ laws of planetary motion. See later: apply to binary systems and extra-solar planet

Both stars in a binary system orbit the center of mass in elliptical orbits with the c of m at one focus. Binary motions enable determination of individual masses of each component.

Most reliable way to determine stellar masses

Law of Gravitation leads to Virial Theorem

From orbital theory for binaries:

total energy of binary orbit = 1/2 time-averaged gravitational potential energy

$E = \langle U \rangle / 2$

Total energy of system $E = -Gm_1m_2/2a$, -ve ≡ bound

Can be shown to apply to all gravitationally bound systems in equilibrium

Since $\langle E \rangle = \langle K \rangle + \langle U \rangle$, where K is the time-averaged kinetic energy

and $-2\langle K \rangle = \langle U \rangle$ expresses virial theorem for a system in equilibrium,

$$\langle E \rangle = \tfrac{1}{2} \langle U \rangle$$

Electromagnetic Radiation and Stellar Properties

From continuous radiation (across all wavelengths) from stars can determine their properties

Treat stars as ideal emitters that absorb all incident radiation and re-radiate over a range of wavelengths

i.e. as Black Bodies that reflect no radiation

From radiant flux in the visible band can measure

Stellar Motions

Distances

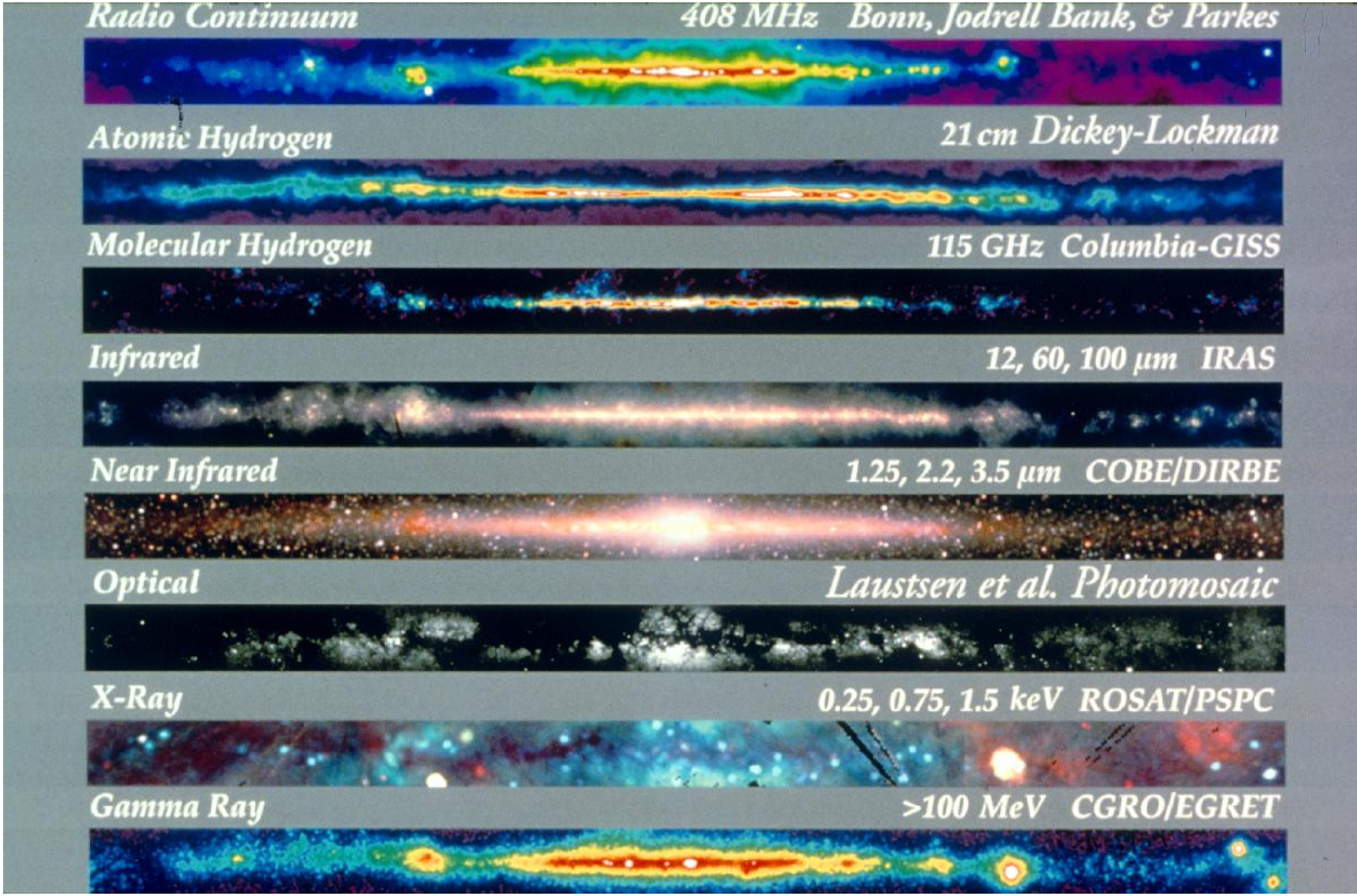
Brightness

"Temperature"

Total flux $F = \int F_\lambda d\lambda$ and brightness $B = L/4\pi d^2$

where L = luminosity (ergs/sec)

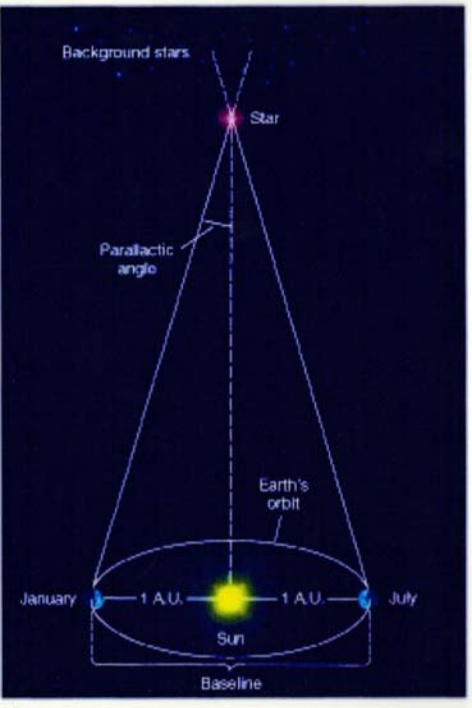
$L_\odot = 3.9 \times 10^{33}$ ergs/sec $= 3.9 \times 10^{26}$ J/sec

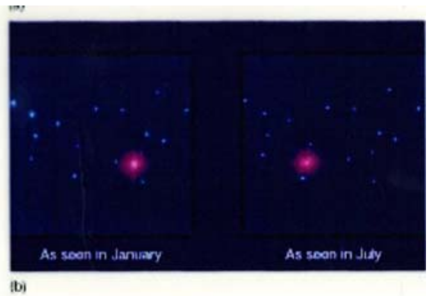


A black body emits a continuous spectrum

Wavelength of peak emission depends on temperature

Distances (nearby stars) from trigonometric parallax





For parallax = 1"
distance = 3.1×10^{18} cm = 1 parsec

Distance = 1/parallax
in parsecs in arcseconds

Parallax – angular shift of nearby star relative to more distant as earth orbits sun

Observations 6 months apart

Baseline = diameter of orbit

Angular change = 2π

Parallax angle = π radians

Distance d = 1AU/tanπ

1 radian = 206265″ (57°.3)

$\therefore$ d= 206265/π AU (π'')

New unit - parallax-second

1 parsec = 1 pc = 206265 AU

= $206265 \times 1.496 \times 10^{13}$ cm

~3×10^{18} cm

1 pc = 3.1×10^{18} cm

Useful fact: at a distance of 1 pc, 1 AU subtends 1″

$\rightarrow$ quick idea of resolution

e.g Taurus star-forming cloud at 150 pc

CARMA resolution 1″ $\therefore$ linear resolution ~ 150 AU

0.5″ ~ 75 AU; 0.7″~ 100 AU

ALMA: 0.01″ ~ 1.5 AU

1 light year = c x t = $3 \times 10^{10} \times 3 \times 10^{7} = 9 \times 10^{17}$ cm

$\therefore$ 1 pc ~ 3×10^{18} cm ~ 3 ly

Parallax measurements tough (parallactic angles small)

First measurement 1838 (Bessel) 61 Cygni - took 4 years

0″.316 $\rightarrow$ d = 1/p″ = 1/0.316 = 3.16 pc

Hipparchos satellite (1989-93): π ~ 0.001″, d up to 1 kpc

400 stars with 1% accuracy; 7000 with 5%

61 Cyg, d = 3.48 pc

<u>Trig parallax distance measures applicable only for stars within solar neighborhood (1 kpc)</u>

Distances to other objects

Convergent point method for stars in clusters

Observe cluster members at two epochs

Parallax for many stars from proper motions and v_R

e.g. Hyades 200 stars, d= 46 pc

Magnitude-distance relationship

use standard candles: variable stars

Spectroscopic parallax

Galaxies: standard candles: variable stars, supernovae

One of a kind proper motion studies:

H_2O masers at Galactic Center 8 kpc

Stellar radiation: the magnitude scale

Brightness = $L/4\pi d^2$ = apparent brightness

Measured in magnitudes (1m brighter than 2m…)

logarithmic scale: each factor 100 in brightness = 5 m

For 2 stars with brightness b_1, b_2, magnitudes m_1, m_2

$$m_1 \propto -\log b_1 \quad m_2 \propto -\log b_2$$

$$\therefore m_1 - m_2 = \text{const} \times \log b_2/b_1 = -5 \text{ for } m_1 = 1 \text{ and } m_2 = 6$$

$$\text{and } b_2/b_1 = 1/100$$

$$\therefore \text{const} = -5/-2 = 2.5$$

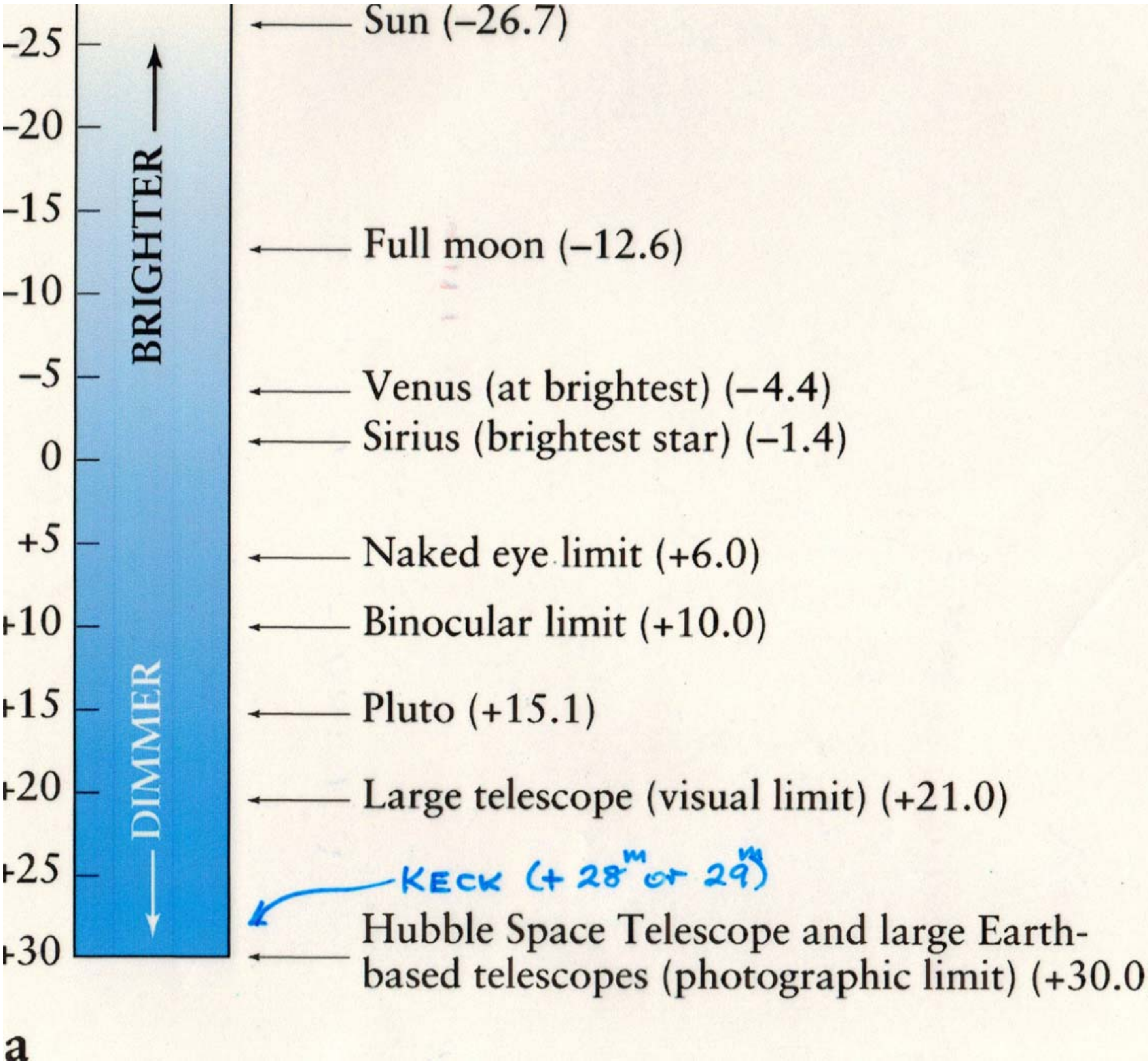
$$\therefore m_1 - m_2 = 2.5 \log b_2/b_1$$

$$\therefore \text{If } \Delta m = 1, \log b_2/b_1 = 0.4$$

$$\therefore \text{And } b_2/b_1 = 2.512$$

$\therefore$ 1st magnitude star 2.512 brighter than 2nd

$(2.512)^2$ brighter than 3rd… 100 x 6th


-25
-20
-15
-10
-5
0
+5
+10
+15
+20
+25
+30
BRIGHTER
DIMMER
Sun (–26.7)
Full moon (–12.6)
Venus (at brightest) (–4.4)
Sirius (brightest star) (–1.4)
Naked eye limit (+6.0)
Binocular limit (+10.0)
Pluto (+15.1)
Large telescope (visual limit) (+21.0)
KECK (+ 28^m or 29^m)
Hubble Space Telescope and large Earth-based telescopes (photographic limit) (+30.0)
a