

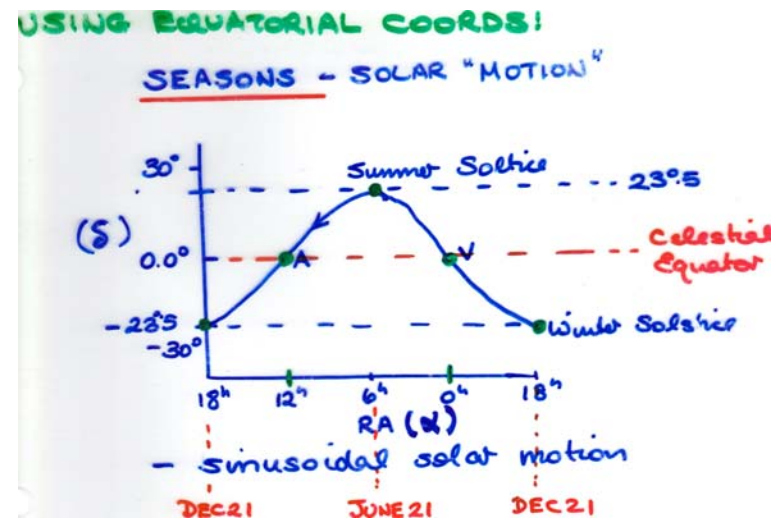
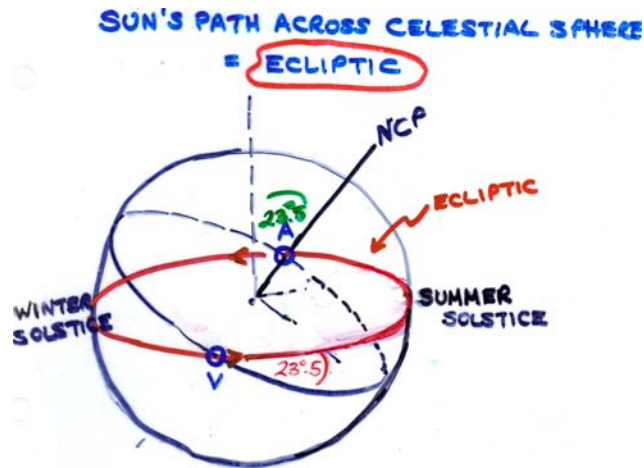
AY 20

Fall 2010

Review - sort of

How do we find objects in the sky consistently?

Equatorial coordinate system (Alt/Az system too "local")



RA, α , measured around celestial equator from first point of Aries, Υ i.e. when Sun on local meridian, $\alpha = 00^h 00^m 00^s$

Dec, δ , measured from celestial equator towards NCP or SCP

Seasons; due to motion of Sun on ecliptic tilted at 23.5°

Sidereal day: time for individual stars to re-cross local meridian

Solar day: time for Sun to re-cross meridian \rightarrow stars rise 4^m earlier each day

Local Sidereal Time (LST) = time since Υ , $00^h 00^m 00^s$, last passed local meridian

Stars best observed near *transit* i.e. when crossing local meridian

e.g. Orion @ $\alpha \sim 5^h$, $\delta \sim -5^\circ$, best observed at LST 5^h = NOW

So we can point. Which instruments' to use?

Radiation at different wavelengths → different information

Wien's law: $\lambda T = \text{constant}$ → different λ s probe different T s

Select instrument based on λ , resolution, collecting power

At optical wavelengths, reflectors preferred over refractors

Angular separation on sky related to linear separation in image plane by plate scale $d\theta/dy = 1/f$; longer focal length → increased "resolution"

but prohibitively long telescopes, limited collecting power

Focal ratio (f/D) $\equiv F$ expressed as f/ratio ; smaller values best Keck $f/1.75$

Achievable angular resolution defined in terms of overlap of Airy diffraction patterns of adjacent sources:

$$\theta_{\min} = 1.22 \lambda/D \equiv \text{Rayleigh Criterion}$$

Light-gathering power of telescopes also important
= illumination = J (energy/sec/area of image) $\propto D^2/f^2$

From $\theta_{\min} = 1.22 \lambda/D$ and $J \propto D^2/f^2$:

larger telescopes \rightarrow better resolution, better illumination

Radio telescopes also parabolic reflectors;

$\theta_{\min} = 1.22 \lambda/D$ demands impossibly large diameter dishes

Have to use interferometer arrays of (not necessarily huge) telescopes

Condition for constructive interference of signals from telescope pairs \rightarrow best resolution $\theta_{\min} = \lambda/D$

Note: here D is largest separation between telescopes
NOT diameter

Typical pattern of astronomy: phenomenological studies, followed by better understanding based on application of fundamental physical principles

e.g. Newton's Laws + Law of Gravitation [$F=Gm_1m_3/r^2$] can be applied to Kepler's laws (empirical)

1. orbit of a planet is an ellipse with the Sun at one focus
2. Equal areas swept out in equal times
3. $P^2 = a^3$ (period in years, semi-major axis in AU)

These become more generalized - apply to binary stars - search for extra-solar planets

1. Sun is seen to be the effective center of mass
2. $dA/dt = L/2\mu$ where L = orbital angular momentum, and μ is reduced mass $m_1m_2/m_1 + m_2$
3. $P^2 = 4\pi^2a^3/G(m_1 + m_2)$

Stellar properties from observables

Direct measurements

Distances (parallax)

Luminosities (U,B,V etc)

Masses (binaries)

Radii (eclipsing binaries,
interferometric
measures)

Rayleigh Criterion

Distance Modulus

Planck Function

Stefan-Boltzmann Law

Kirchkoff's Laws

Maxwell-Boltzmann Distribution

Boltzmann Equation

Saha Equation

Using stellar spectra:

Spectral type $\equiv T_{\text{eff}}$

Luminosity classes \equiv gravity,
pressure, density

Radial velocities, z

Position on HR Diagram:

Stellar radii

Distances (spectroscopic parallax)

main-sequence masses
(approx)

Ages of stars (later)

Spectra also provide info on:

Rotational velocities

Chemical abundances

Magnetic fields

Mass inflow-outflow

Treating stars as black bodies and measuring radiant flux → stellar properties: luminosity, mass, distance, temperature, evolutionary state

Observed brightness $B = L/4\pi d^2 =$ radiant flux F

measured in magnitudes - a logarithmic scale, L = luminosity

$m_1 - m_2 = 2.5 \log b_2 / b_1$ where m is apparent magnitude

Defining M , absolute magnitude = magnitude at 10pc

→ $m - M = 5 \log d - 5$ and eventually $m - M_v = 5 \log d - 5 + A_v$

→ distance measurement if M is known

L, M are intrinsic to star; m, F depend on distance

Distances to nearby stars from trig parallax measures

$d = 1/\pi$ where π is parallax angle in arcseconds

Distance to star with parallax $1'' = 1$ parsec = 206265 AU = 3×10^{18} cm

Beyond ~ 1 kpc spectroscopic parallax method sometimes used

spectral type of star indicates M_v

At greater distances main sequence fitting methods using clusters,

Even more distant - RR Lyrae variables, Cepheid variables (recall period luminosity relation)

Masses of stars can be determined in binary systems

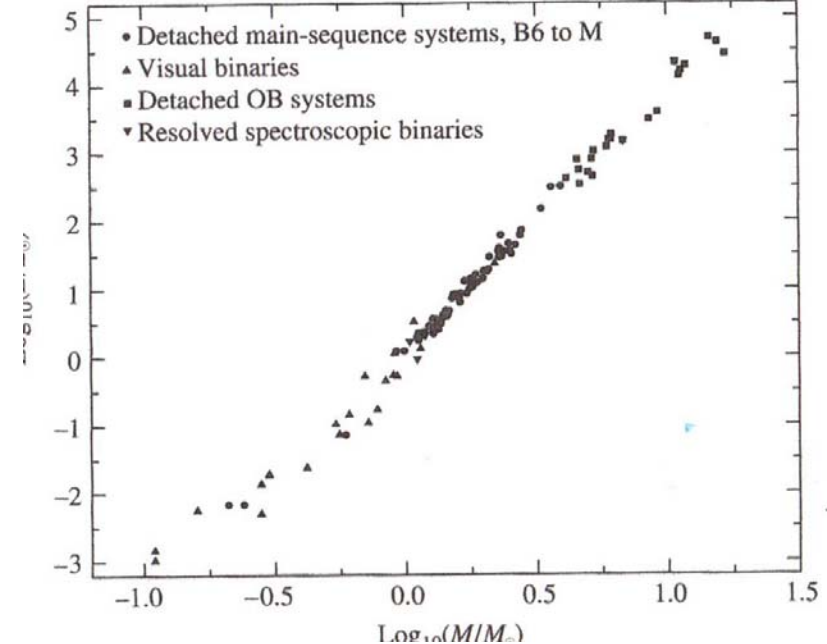
Masses of binary stars define empirical mass-luminosity relation

For $M > 3 M_{\odot}$, $L \propto M_{\star}^3$

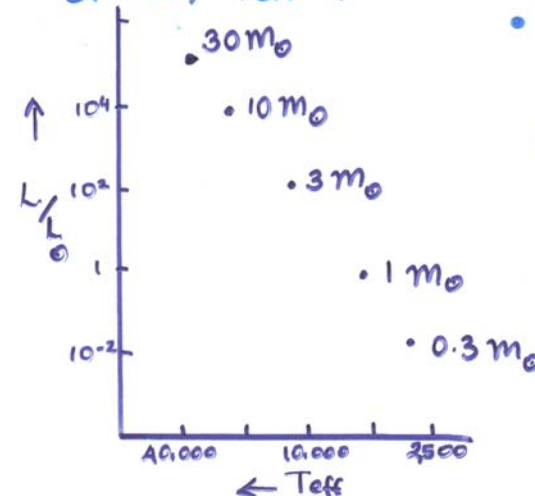
For $M < 0.5 M_{\odot}$, $L \propto M_{\star}^{2.5}$

VERY approximate

Plotting stars of known mass on H-R diagram (L v. T_{eff}) also instructive



H-R DIAGRAM IN TERMS OF L , T_{eff} ↓



• PLOTTING L v T_{eff} FOR KNOWN MAIN SEQUENCE BINARIES (MASSES DERIVABLE)



EARLY TYPE STARS HOTTER, & MORE MASSIVE

Treating stars as black bodies and measuring radiant flux → stellar properties: luminosity, mass, distance, temperature, evolutionary state (continued)

Stellar masses from stars in binary systems

Temperatures: Black body radiation has a characteristic shape:
Stellar temperatures (roughly) from Wien law (empirical)

$\lambda_{\max} T = 0.29 \text{ K } (\lambda \text{ cm})$ peak of curve → surface temp

Or Stefan-Boltzmann equation $L = A\sigma T^4$ and $F \propto 1/d^2$

lead to $F_{\text{surface}} = \sigma T_{\text{eff}}^4$ (also empirical)

Nevertheless, explaining in terms of basic physics difficult

Rayleigh-Jeans law: $B_{\lambda}(T) \approx 2ckT/\lambda^4$ for long λ

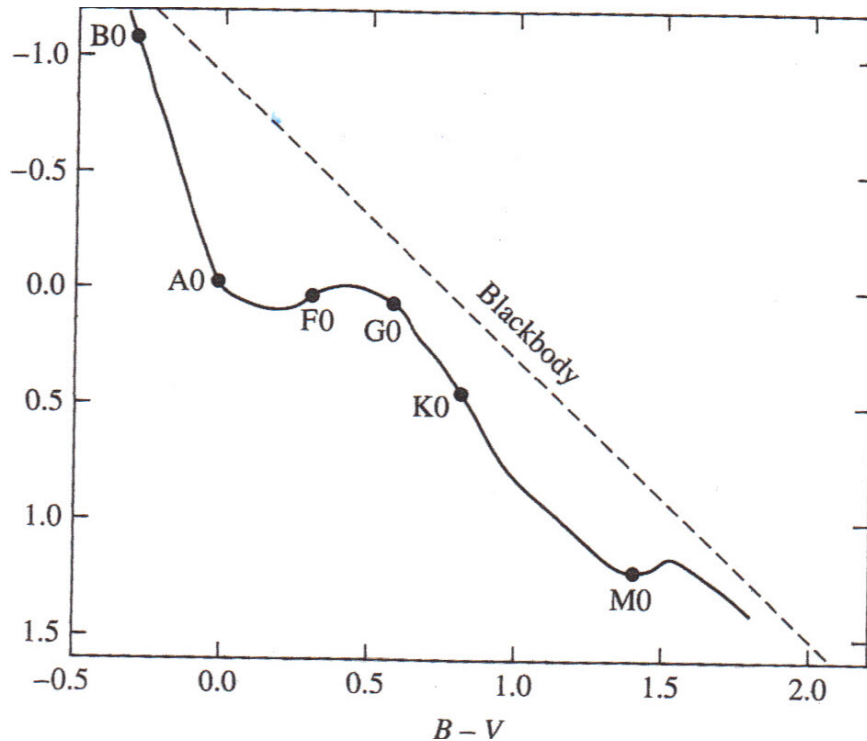
Wien's law: $B_{\lambda}(T) \approx a\lambda^{-5}e^{-b/\lambda kT}$ for short λ

Planck's interpretation most effective → Planck function

$$B_{\lambda}(T) = 2h \frac{c^2}{\lambda^5} \left(e^{hc/\lambda kT} - 1 \right)^{-1}$$

$$B_{\nu}(T) = 2h \frac{\nu^3}{c^2} \left(e^{h\nu/kT} - 1 \right)^{-1}$$

Measuring flux in different bands across Planck function → color-color Diagram and indicates physical properties



Based on color indices B-V etc

$$=M_B - M_V = m_B - m_V$$

i.e. an intrinsic stellar property

Indicates real stars don't behave like B-Bs

Hottest stars behave most like black bodies

Absorption of radiation displaces stars from B-B line

In any case, not measuring total radiation. To correct introduce **bolometric correction**

$$BC = m_{bol} - V = M_{bol} - M_V$$

Usual pattern:Kirkchoff's Laws → empirical definitions

Atomic theory needed to explain spectral lines

Briefly: hot dense gas (or opaque solid) emits continuous spectrum of radiation as described by Planck function

Hot diffuse (low density) gas produces bright emission lines when an electron makes a downward transition from higher to lower orbit.
Energy lost = $h\nu$ or hc/λ .

Cool diffuse gas in front of a source of continuous radiation produces dark absorption lines in continuous spectrum when an electron makes an upward transition to higher orbit.

Enables spectral classification of stars - essentially a temperature scale

Balmer lines peak in strength at AO, $T_e = 9250\text{K}$

at lower temperatures harder to excite hydrogen

at higher temperatures ionization is beginning

HeI lines most intense in B2 stars $T_e = 22,000\text{K}$

CII lines most intense in KO stars $T_e = 5250\text{K}$

Spectra show peak of Planck function moving to shorter λ as T_e increases

Lead to **Hertzsprung Russell diagram**

Needed: a physical basis for spectral classification

For each element:

What determines relative numbers of atoms in each excitation state?

What determines relative numbers of atoms in each ionization state?

Maxwell-Boltzmann Distribution Function: $n_v dv = n \left(\frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} 4\pi v^2 dv$

Boltzmann equation: $N_b/N_a = g_b e^{-E_b/kT} / g_a e^{-E_a/kT} = g_b/g_a e^{-(E_b-E_a)/kT}$

Saha Equation: $\frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{n_e Z_i} \left(\frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{-\chi_i/kT}$

Finally $N_2/N_{\text{total}} = N_2 \times (N_1 + N_2)^{-1} \times N_{\text{I}}/N_{\text{total}}$ (can assume $N_{\text{I}} = N_1 + N_2$)
 $= N_2/N_1 (1 + N_2/N_1)^{-1} \times (1 + N_{\text{II}}/N_{\text{I}})^{-1}$

- Most stars have similar relative abundances of elements as Sun
- Dominated by H, He, and then "metals"
- Spectral line intensities strongly dependent on T

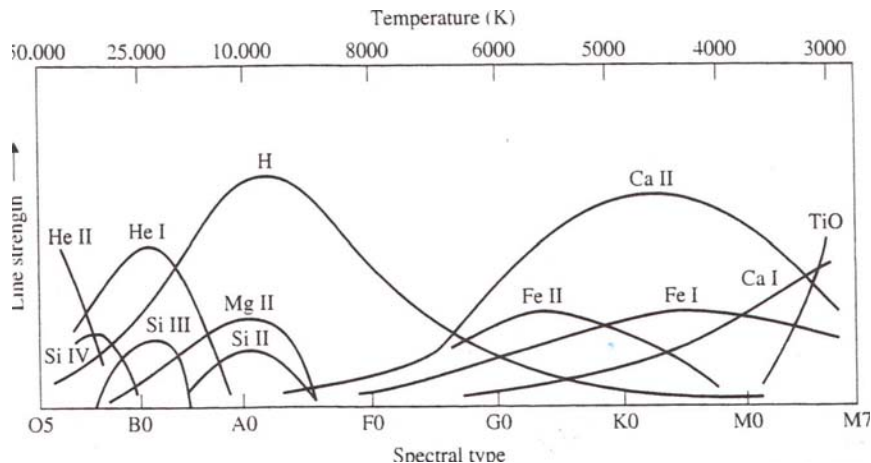


TABLE 9.2 The Most Abundant Elements in the Solar Photosphere. The relative abundance of an element is given by $\log_{10}(N_{el}/N_H) + 12$. (Data from Grevesse and Sauval, *Space Science Reviews*, 85, 161, 1998.)

Element	Atomic Number	Log Relative Abundance
Hydrogen	1	12.00
Helium	2	10.93 ± 0.004
Oxygen	8	8.83 ± 0.06
Carbon	6	8.52 ± 0.06
Neon	10	8.08 ± 0.06
Nitrogen	7	7.92 ± 0.06
Magnesium	12	7.58 ± 0.05
Silicon	14	7.55 ± 0.05
Iron	26	7.50 ± 0.05
Sulfur	16	7.33 ± 0.11
Aluminum	13	6.47 ± 0.07
Argon	18	6.40 ± 0.06
Calcium	20	6.36 ± 0.02
Sodium	11	6.33 ± 0.03
Nickel	28	6.25 ± 0.04

LATER: fractions by weight
 H X
 He Y
 Metals Z
 0.70 0.28 0.02

³²Details of the construction of a model star will be deferred to Chapter 10.

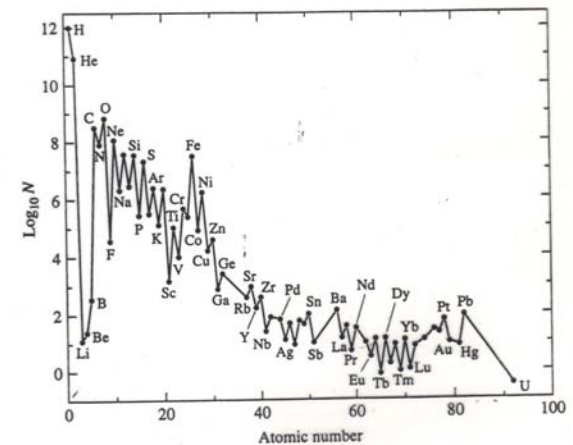


FIGURE 15.16 The relative abundances of elements in the Sun's photosphere. All abundances are normalized relative to 10^{12} hydrogen atoms. (Data from Grevesse and Sauval, *Space Sci. Rev.*, 85, 161, 1998.)

Stellar Properties from the Hertzsprung-Russell diagram

$$L_{\star} = 4\pi R_{\star}^2 F = 4\pi R_{\star}^2 \sigma T_e^4$$

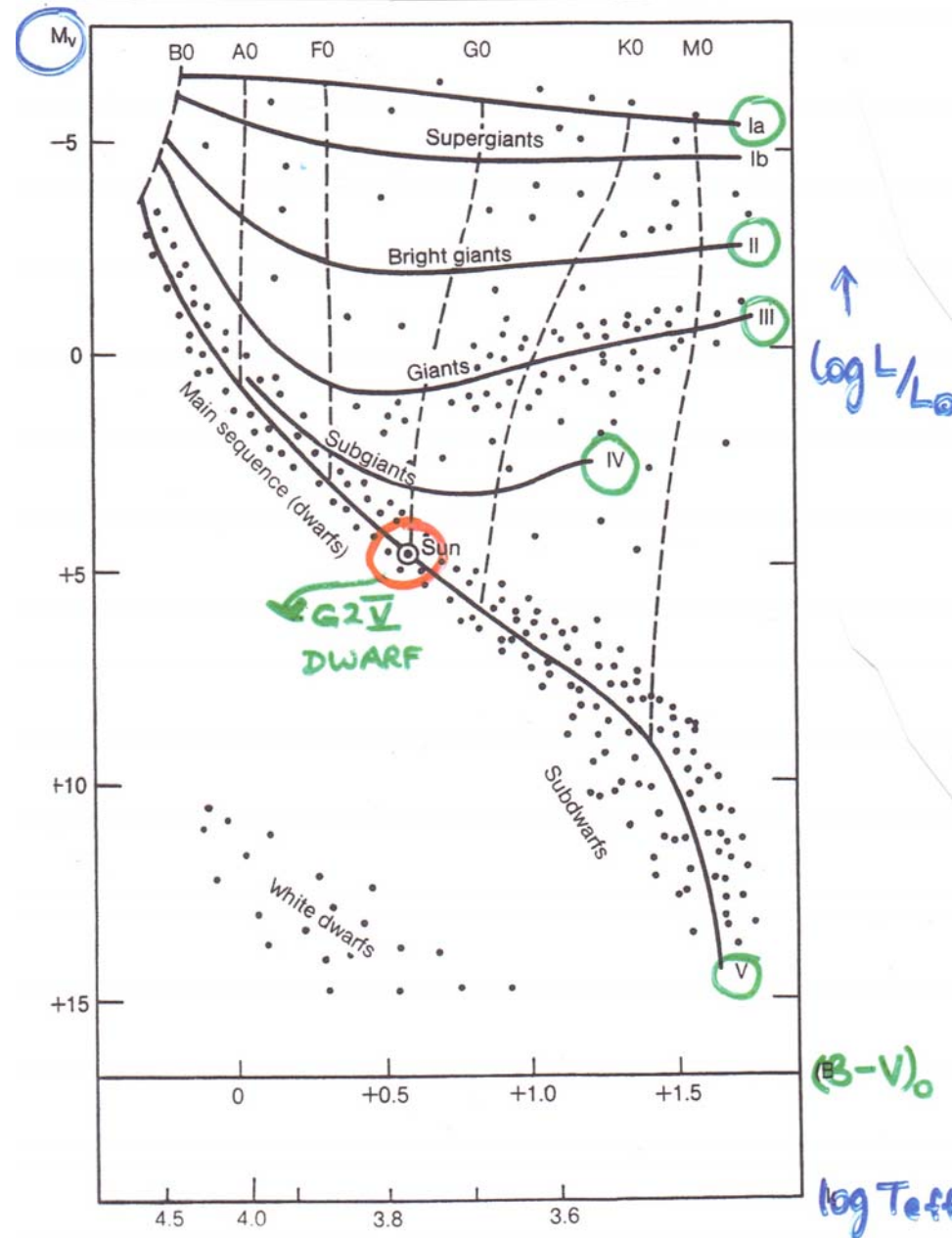
→ absolute magnitude of stars of same spectral type varies with R_{\star}

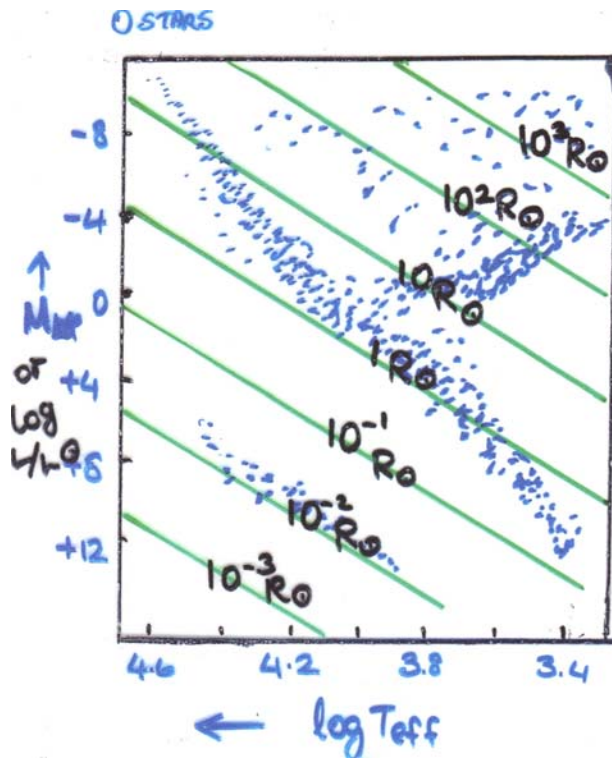
$$R_{\star} = \frac{1}{T_{eff}^2} \sqrt{\frac{L}{4\pi\sigma}}$$

for fixed R_{\star} , $\log L_{\star} \propto \log T_{eff}$

→ lines of constant R_{\star} in H-R diagram

Express R_{\star} , T_{\star} , L_{\star} , in terms of R_{\odot} , T_{\odot} , L_{\odot} ,



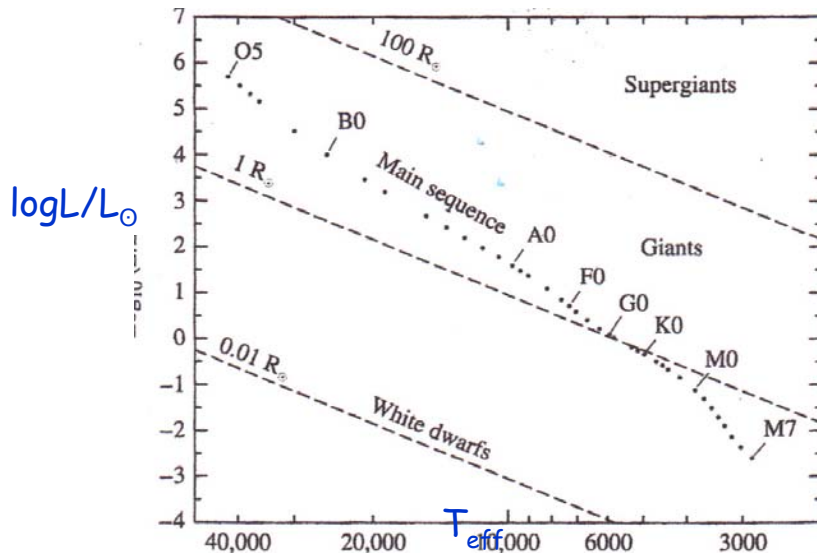


- spectral type + luminosity class (class indicates line width) $\rightarrow M_v$
distance from $m-M = 5 \log d - 5$

- stellar density, $\rho_* = \frac{M_*}{\frac{4}{3} \pi R_*^3}$
varies with position in H-R diagram

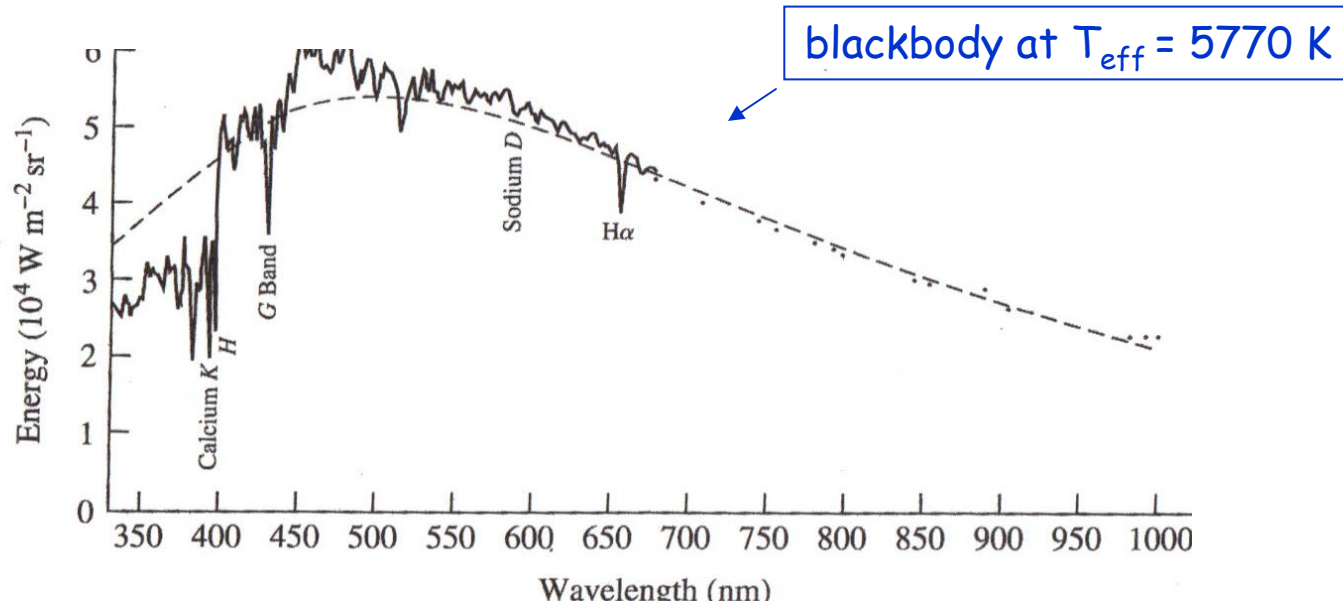
radii from $L + T_{\text{eff}}$, hence density if masses known (e.g. from binaries)

<u>white dwarfs</u>	<u>supergiants</u>
$R \sim 0.001 R_{\odot}$	$R \sim \text{few} \times 10^3 R_{\odot}$
$\rho \sim 10^9 \text{ kg/m}^3$	$\rho \sim 10^{-4} \text{ kg/m}^3$



Eventually saw that position of star on main-sequence depends on its mass

Stars are not blackbodies



Sun is clearly not a blackbody

Spectral lines impact continuous spectrum of emission

Line blanketing by dense pattern of metal absorption lines

(emission lines in UV or X-ray bands possible)

Absorption effects \equiv "opacity" effects

Larger implication: T_{eff} from $F_{\text{surface}} = \sigma T_{\text{eff}}^4$ is not *true photospheric temperature*

To compensate (and understand transfer of radiation) introduce concept of Local Thermodynamic Equilibrium

Define a "local box" over which temperature remains "constant"
i.e. distance over which T changes \gg mean free path of ptcles/photons
→ T_{ex} (Boltzmann), T_i (Saha), T_{kin} (Max-Boltz), T_{color} (Planck fuⁿ) equal
and thermodynamic equilibrium applies within "box"
(note: T_{eff} applies to specific levels in star)

To determine how radiation traverses star assume LTE and define
absorption as any process that removes photons from beam

Decrease in intensity of beam = $dI_\lambda = -\kappa_\lambda I_\lambda \rho ds$, (κ_λ is absorption coefft = opacity)

$$I_\lambda = I_{\lambda,0} e^{-\kappa_\lambda \rho ds}$$

∴ For pure absorption, intensity falls off exponentially i.e. by factor e^{-1}
at characteristic distance $\ell = 1/\kappa_\lambda \rho$

define optical depth $d\tau_\lambda = -\kappa_\lambda \rho ds$ (s measured in direction of photon's
motion i.e. at stellar surface $\tau_\lambda = 0$)

Hence τ_λ and I in terms of τ_λ

$$dI_\lambda = -\kappa_\lambda I_\lambda \rho ds \rightarrow I_\lambda = I_{\lambda,0} e^{-\int \kappa_\lambda \rho ds}$$

pure absorption: I falls off by e^{-1} at characteristic distance $\ell = 1/\kappa_\lambda \rho$

scattering: $\ell = \text{photon mean free path} = 1/n\sigma_\lambda = 1/\kappa_\lambda \rho$

$$d\tau_\lambda = -\kappa_\lambda \rho ds; \therefore I_\lambda = I_{\lambda,0} e^{-\tau_\lambda}$$

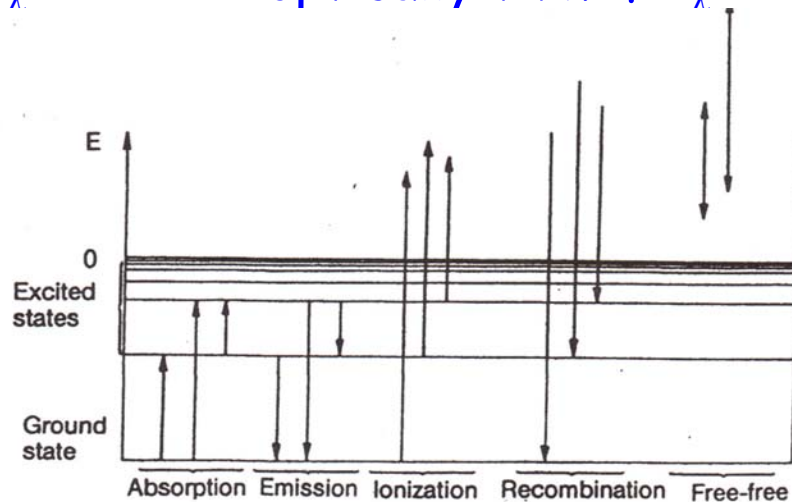
s in direction of photon motion -outward; τ_λ inward from surface

Since $\tau_\lambda = \kappa_\lambda \rho \int ds = \int ds/\ell$, optical depth = number of mean free paths from original position to surface

optically thick if $\tau_\lambda \gg 1$ optically thin if $\tau_\lambda \ll 1$

Sour

Fig. 5.2. Different kinds of transitions between energy levels. Absorption and emission occur between two bound states, whereas ionization and recombination occur between a bound and a free state. Interaction of an atom with a free electron can result in a free-free transition

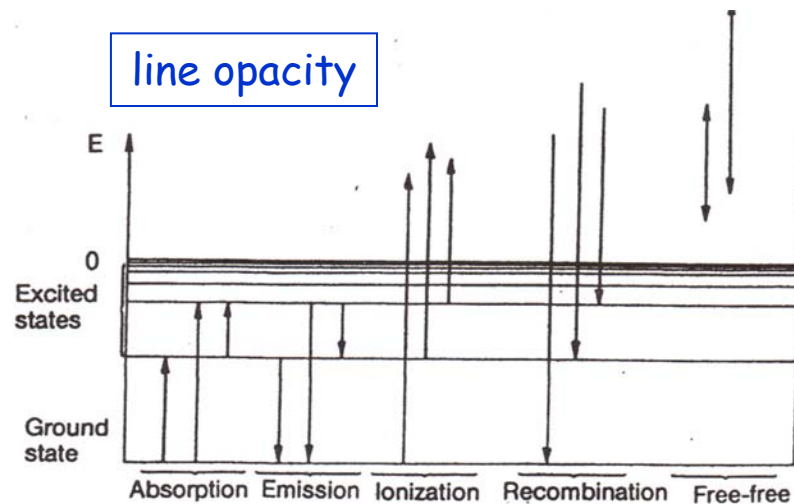


Sources of Opacity: slowly varying affects continuum; rapid variations → dark spectral lines

1. bound-bound transitions: photons "lost" to beam at discrete λ s
2. free-free transitions: absorption & bremsstrahlung - no preferred λ
3. bound-free transitions: photoionization* - any photon w. $\lambda < hc/\chi$
4. electron scattering:

* photoionization of H⁻ ions important continuum opacity source in stars cooler than F0
B and A stars: continuum opacity from photoioniz. of H atoms or free-free absorption
O stars: electron scattering and photoionization of He

Fig. 5.2. Different kinds of transitions between energy levels. Absorption and emission occur between two bound states, whereas ionization and recombination occur between a bound and a free state. Interaction of an atom with a free electron can result in a free-free transition



continuum opacity

5.2
"the H atom"

Real Radiative Transfer (not just removing photons from beam)

- For stars, T and P_{rad} decrease outwards; $P_{\text{rad}} = 4\sigma T^4/3c$
→ net flow of photons outwards
- introduce **emission coefficient** j_λ , analogous to κ_λ
 $dI_\lambda = -\kappa_\lambda \rho I_\lambda ds + j_\lambda \rho ds$ and $j_\lambda / \kappa_\lambda = S_\lambda$, **source function**
 $1/\kappa_\lambda \rho \times dI_\lambda/ds = I_\lambda - S_\lambda$ transfer equation*
in LTE, for optically thick gas, $S_\lambda = B_\lambda$
- a plane parallel, **gray**, atmosphere → **$\cos\theta dI_\lambda/d\tau_{\lambda,v} = I - S$**
 $\tau_{\lambda,v}$ is vertical optical depth, *independent* of direction of "ray"
- expressing in terms of P_{rad} , F_{rad} :
 $dP_{\text{rad}}/d\tau_v = F_{\text{rad}}/c$ or **$dP_{\text{rad}}/dr = -\kappa \rho F_{\text{rad}}/c$** ; (Rosseland mean κ)
integrating $dP_{\text{rad}}/d\tau_v$ with $P_{\text{rad}} = 4\pi/3c\langle I \rangle$ and Eddington approx
 $T^4 = \frac{3}{4}T_{\text{eff}}^4(\tau_v + 2/3)$, $T = T_{\text{eff}}$ at $\tau_v = 2/3$
 \therefore photosphere at $\tau_v = 2/3$ (width $\sim 1\% R_*$)

Note: for Sun, $T_{\text{eff}} = 5777\text{K}$. At $\tau_v = 0$, $T = 4852\text{K}$,

Equations of Stellar Structure

Describe how star “works” assuming equilibrium

Validity can be tested:

- observable properties should match those computed from models based on structure equations

The equations govern:

- the variation in pressure with radius in the stellar interior
(equation of hydrostatic equilibrium)
- the distribution of mass
(equation of continuity - or mass conservation)
- the production of energy
(energy conservation equation)
- the transport of energy
(variation of temperature as a function of radius; depends on way energy is transported - by radiation, convection, or conduction)

Important questions

- What is mean molecular weight, μ ? (1)
- What determines ε = total energy released /gm/sec? (2)
 - nuclear burning (fusion) can sustain observed luminosity of Sun for $> 10^{10}$ years
 - Temperature required for fusion $\sim 10^7$ K (if quantum tunneling allowed) \approx central temp of Sun
 - and $\varepsilon = \varepsilon_{\text{nuclear}} + \varepsilon_{\text{gravity}}$
- How is energy transported and how does that affect temperature structure? (3)

Mean molecular weight of gas $\mu = \langle m \rangle / m_H$ (1)

$$\frac{1}{\mu_n} = \sum_j \frac{X_j}{A_j} \quad \text{for completely neutral gas}$$

$$\frac{1}{\mu_i} = \sum_j \frac{(1 + z_j) X_j}{A_j} \quad \text{for completely ionized gas}$$

Where X_j = mass fraction for atoms of type j ; $A_j = m_j / m_H$

Usual usage: X = total mass of H / total mass of gas

Y = total mass of He / total mass of gas

Z = total mass of metals / total mass of gas

Typically abundances are solar: $X = 0.70$ $Y = 0.28$ $Z = 0.02$

$$1/\mu_n = X + \frac{1}{4}Y + \langle 1/A \rangle_n Z$$

$$1/\mu_i = 2X + \frac{3}{4}Y + \langle (1+z)/A \rangle_i Z \quad (z_j = \# \text{ free electrons released by atom } j)$$

$$1/\mu_i = 2X + \frac{3}{4}Y + \frac{1}{2}Z$$

Transport of Energy (3)

a) Conduction, b) Radiative Energy Transport, c) Convection

Conduction not very effective in most stars

Radiative Transport - energy from nuclear processes (2)

- net flow of photons to surface impacted by opacity

Convection - hot buoyant mass elements move outwards, cooler fall inwards

Distinguish between b) and c) thru condition for convection

$$\left| dT/dr \right|_{\text{act}} > \left| dT/dr \right|_{\text{ad}}$$

actual temperature gradient *superadiabatic* for constant μ

Express also as: $d \ln P / d \ln T < \gamma / (\gamma - 1) = 2.5$ for monatomic gas

For $d \ln P / d \ln T < 2.5$, convective transport

For $d \ln P / d \ln T > 2.5$ radiative transport

Energy Sources (2)

Nuclear reactions produce sufficient energy to sustain luminosity over stellar lifetimes. Dominant processes depend on stellar temperatures, masses, evolutionary state (composition)

Equations of stellar structure

- equation of hydrostatic equilibrium
$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2}$$
 - mass conservation equation
$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$
 - energy conservation equation
$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$
 - radiation transport
$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\bar{\kappa} \rho}{T^3} \frac{L_r}{4\pi r^2}$$
 - adiabatic convection
$$\frac{dT}{dr} = -\left(1 - \frac{1}{\gamma}\right) \frac{\mu m_H}{k} \frac{GM_r}{r^2}$$
- $$P = P_{\text{gas}} + P_{\text{radiation}} = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4$$

- For modelling, have to supplement structure equations with constitutive relations - expressions for ρ , κ , ϵ , in terms of properties of local material P , T , composition
- Also have to impose boundary conditions
- As $r \rightarrow R_*$ (surface). $T \rightarrow 0$, $\rho \rightarrow 0$, $P \rightarrow 0$
- As $r \rightarrow 0$ (center) , $M_r \rightarrow 0$. $L_r \rightarrow 0$
- All parameters are inter-related: P , $\langle \kappa \rangle$, ϵ_{pp} (ϵ_{CNO} etc) depend on local composition, ρ , T
- Vogt Russell "theorem" - once mass and composition structure defined, the radius-luminosity combination is defined, and also subsequent evolution.
- Effectively, evolution determined by composition changes due to nuclear burning
- Implications; Hydrogen burning stars lie on main sequence - a mass sequence; Eddington Limit constrains mass? Sun, Standard Model, provide great tests for theory

Effects of the Interstellar Medium

Distance modulus changes $m - M = 5 \log d \text{ (pc)} - 5 + A_\lambda \text{ (mag)}$

change of magnitude \approx optical depth in line of sight: $A_\lambda \approx \tau_\lambda$

$A_\lambda \approx \tau_\lambda = \sigma_\lambda N_d$, where N_d = column density of dust particles

Extinction \propto number of grains in line of sight if σ_λ constant

Mie scattering: when particle size \sim wavelength of radiation,
 $\sigma_\lambda \propto 1/\lambda$

optical radiation is reddened since blue light scattered preferentially

reddening distorts T_e , distance modulus, color-color diagram etc

Correct using color excess: $E_{B-V} = (B-V)_{\text{measured}} - (B-V)_0$

Empirically, $E_{B-V} = 3A_V$

Also gas in ISM: mostly HI, HII, or H₂ (70% H, 28% He, 2% metals)

gas, dust well-mixed: $n_{\text{dust}} \sim 10^{-13}/\text{cm}^3$, $\langle n_{\text{gas}} \rangle \sim 1/\text{cm}^3$

mass $\sim 100 \times$ dust mass

dust opacity/unit mass \gg gas opacity/unit mass

Five phases of interstellar gas

	T(K)	n (cm ⁻³)
Very cool molecular clouds (mostly H ₂)	20	$> 10^3$
Cool clouds (mostly HI)	100	20
Warm neutral gas envelopes of cool clouds	6000	0.5-0.3
Hot ionized gas (HII regions)	8000	> 0.5
Very hot diffuse ionized "coronal" gas ionized, heated by supernovae explosions	10^6	10^{-3}

Manifestations of Star Formation in ISM

HII Regions: simplistically: boundary from balancing ionization and recombination in gas around massive young star → Stromgren sphere

Central stars, $T \sim 3 \times 10^4$; Wien $\rightarrow \lambda_{\text{peak}} < 912 \text{ \AA}$, very blue

HII region; $n = 3 \rightarrow 2$ transition dominates = H α (6565 \AA)

\therefore emission from HII regions is red

Ionization / recombination balance: $N_*/4/3\pi r^3 = \alpha n_e^2$

$$R_S = (3N_*/4\pi\alpha)^{1/3} \times n_e^{-2/3}$$

Collapsing Cores \rightarrow new young stars: Virial Theorem $2K + U = 0$

Jeans mass

$$M_J = \left(\frac{5kT}{G\mu m_H} \right)^{3/2} \left(\frac{3}{4\pi\rho_0} \right)^{1/2}$$

Jeans length

$$R_J = \left(\frac{15kT}{4\pi G\mu m_H \rho_0} \right)^{1/2}$$

Various timescales for stellar evolution

$$\text{Free fall time} = t_{\text{ff}} = [3\pi/32 \times 1G\rho_0]^{1/2}$$

core in approximate hydrostatic equilibrium

$$\begin{aligned}\text{Kelvin-Helmholtz } t_{\text{KH}} &= \text{thermal energy only/Luminosity} \\ &= 3/10 GM_*^2/R_*L_*\end{aligned}$$

$$\begin{aligned}t_{\text{nuclear}} &= \text{time to radiate all energy from fusion} \\ &= M_*C^2/L_*\end{aligned}$$

Nuclear reactions change μ

$$\text{Stellar evolution } t_{\text{ff}} \rightarrow t_{\text{KH}} \rightarrow t_{\text{nuclear}}$$

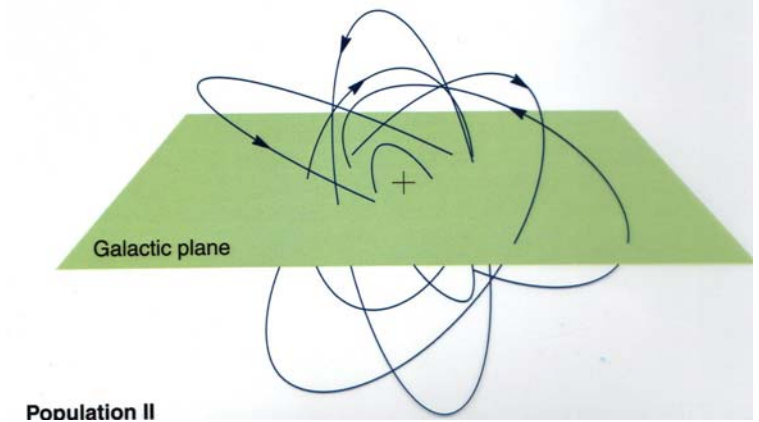
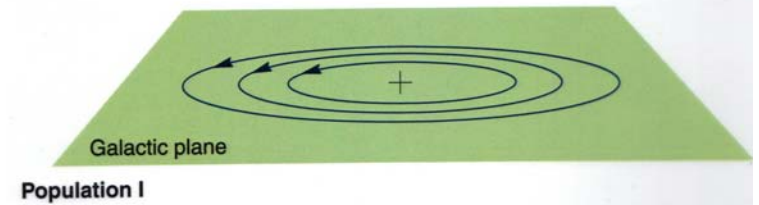
Hydrostatic core Hayashi H-burning...

track to

main-seq

- Evolution & Nuclear burning a function of mass etc.

Orbits of disk stars and halo stars



3 populations of stars in our Galaxy

Initially Pop I and Pop II only;
kinematically different

Pop I star velocities relative to Sun much
lower than Pop II velocities

Pop I stars mainly in disk of galaxy; Pop
II well away from plane

Later, metallicity differences noted:

Pop I metal rich, Z up to 0.03 ; Pop II
metal poor, $Z > 0$

Theorists postulate Pop III:

Pop III: first stars after Big Bang; no
metals $Z=0$

Open clusters

loosely bound

$T_{\text{evap}} \sim 10^8$ yrs

younger

metal rich

Pop I

(galactic clusters)

Globular clusters

gravitationally bound

$T_{\text{evap}} \sim 10^{11}$ yrs

typically old

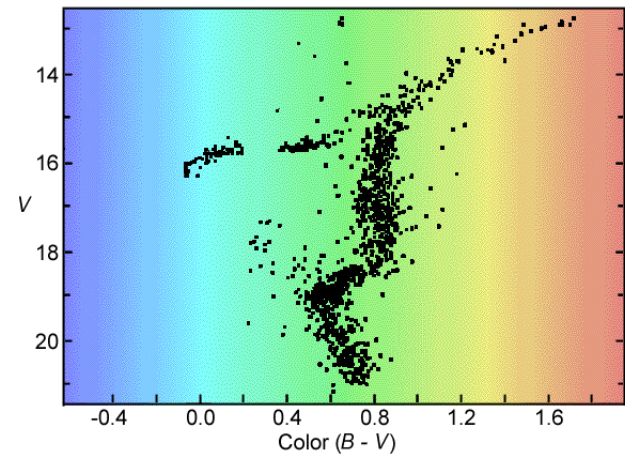
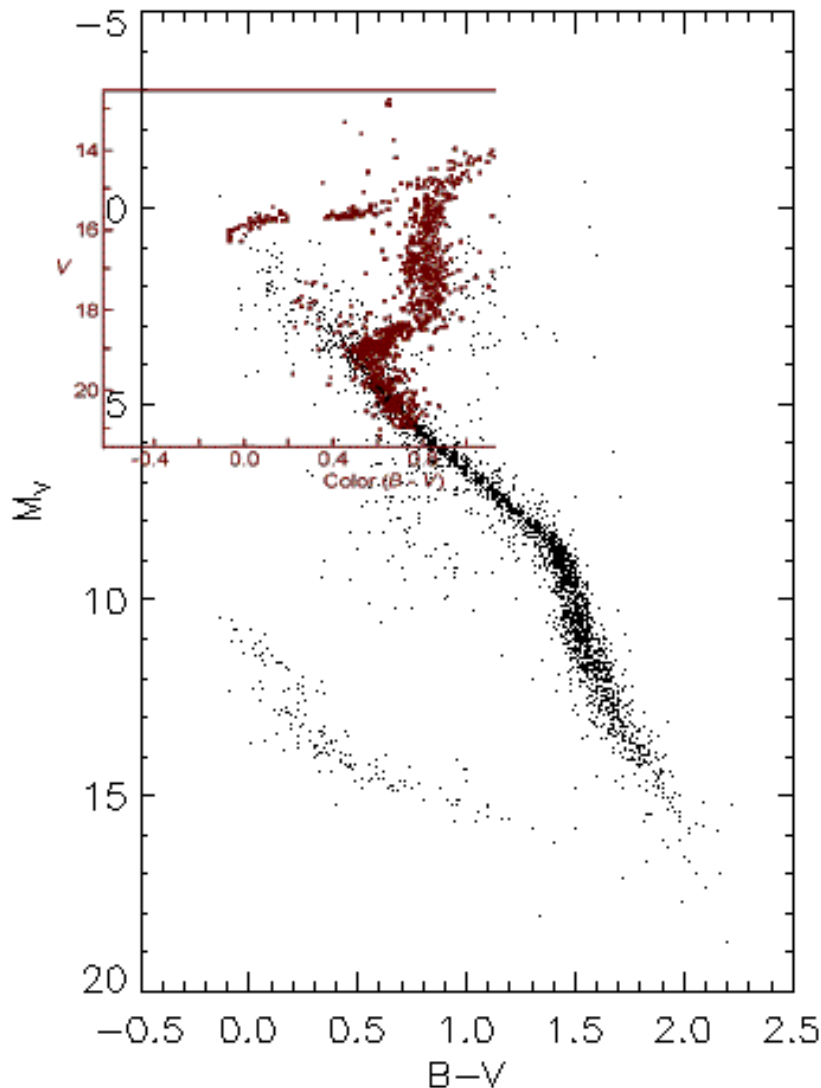
metal poor

Pop II

Distance scales from:
main sequence fitting
RR Lyrae variables*
Cepheid variables

*RR Lyrae stars fainter: use for
smaller distance scales than Cepheids

M3 in Canes Venatici - $\sim 0.5 \times 10^6$ stars; many variables
(RA 13h 44m Dec 28°11^h)



Age = 2.6×10^9 yrs (Sandage)

Vertical shift $\sim 19 - 3.5 = 15.5$ (15.4)

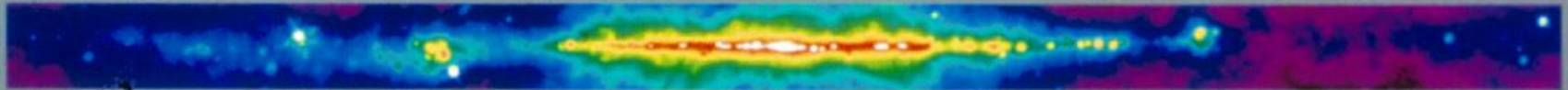
$$= m_v - M_v = 5 \log d - 5$$

$$\therefore \log d = (15.4 + 5)/5 = 20.4/5 = 4.1$$

$$\therefore d = 12 \text{ kpc}$$

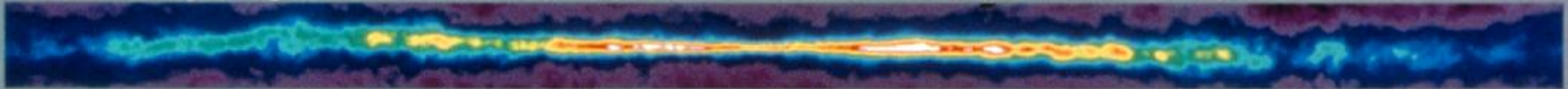
Radio Continuum

408 MHz Bonn, Jodrell Bank, & Parkes



Atomic Hydrogen

21 cm Dickey-Lockman



Molecular Hydrogen

115 GHz Columbia-GISS



Infrared

12, 60, 100 μ m IRAS



Near Infrared

1.25, 2.2, 3.5 μ m COBE/DIRBE



Optical

Laustsen et al. Photomosaic



X-Ray

0.25, 0.75, 1.5 keV ROSAT/PSPC



Gamma Ray

>100 MeV CGRO/EGRET



The Rotation of the Milky Way

- Flatness of MW suggests rotation about axis perpendicular to plane
- Observations of stars, gas confirm **differential** rotation
- i.e. Milky Way does not rotate like a rigid body
- angular velocity depends on distance from GC
- Observable effects of galactic rotation derived by Jan Oort
- Sun shares in differential rotation - has to be taken into account

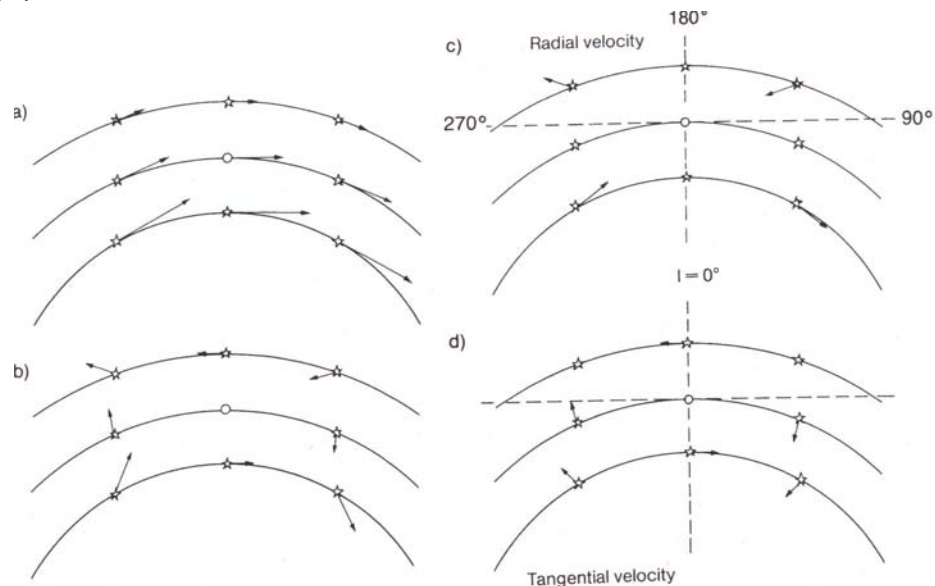


Fig. 18.13a–d. The effect of differential rotation on the radial velocities and proper motions of stars. (a) Near the Sun the orbital velocities of stars decrease outwards in the Galaxy. (b) The relative velocity with respect to the Sun is obtained by subtracting the solar velocity from the velocity vectors in (a). (c) The radial components of the velocities with respect to the Sun. This component vanishes for stars on the same orbit as the Sun. (d) The tangential components of the velocities

$$v_r = A \sin 2l$$

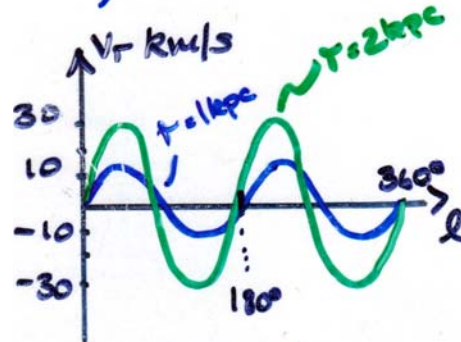
$$v_t = A \cos 2l + B$$

$$\mu = A \cos 2l + B$$

OBSERVED v_r FOR
STARS AT SAME l →
DOUBLE SINE CURVE
WITH AMPLITUDE A

Hence A if r known

OBSERVED μ → CURVE
OFFSET - B



FOR STARS AT SAME DIST r
 v_r AS FUNCTION OF l

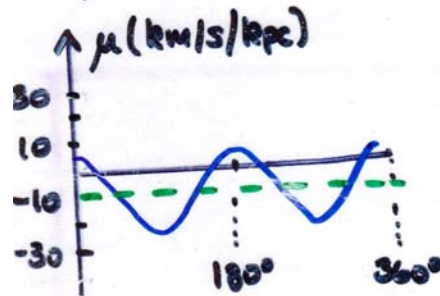
AMPLITUDE $\Rightarrow A$
 $= 15 \text{ km s}^{-1} \text{ kpc}^{-1}$
HIPPARCOS $\rightarrow 14.8 \pm 0.8$

μ INDEPENDENT OF
DISTANCE

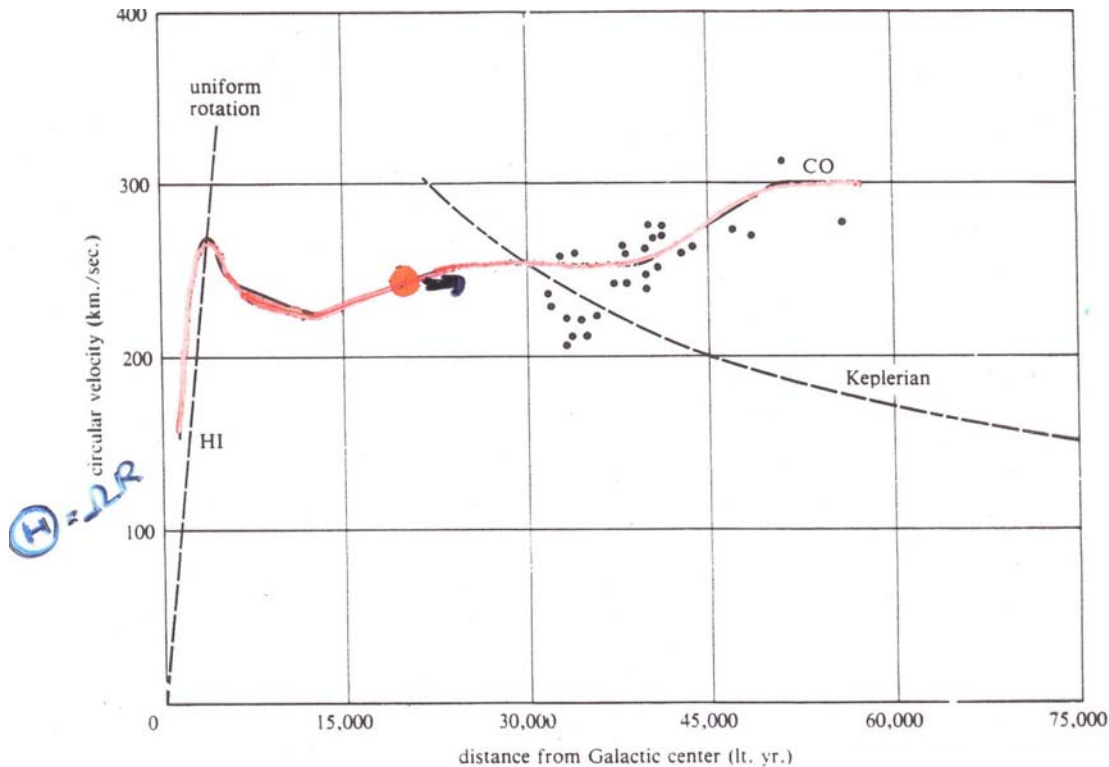
μ AS FUNCTION OF l

AMPLITUDE $\Rightarrow A$

"MEAN" $\Rightarrow B$
 $= -10 \text{ km s}^{-1} \text{ kpc}^{-1}$
HIPPARCOS $\rightarrow -12.4 \pm 0.6$

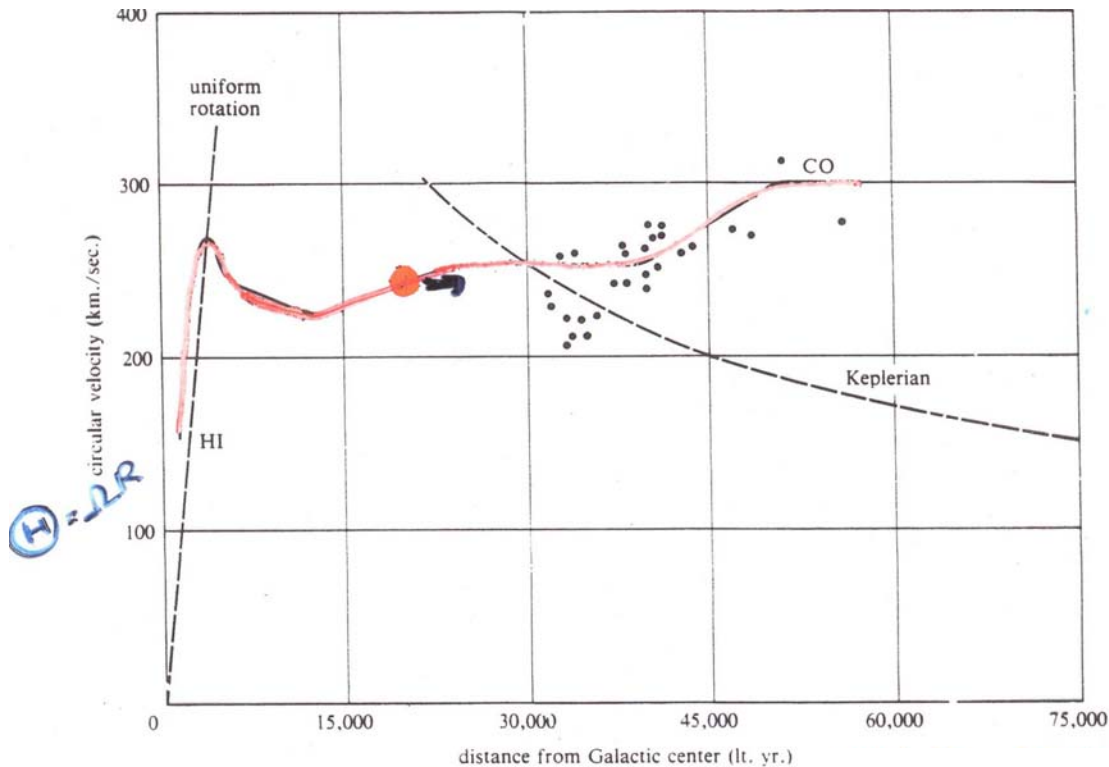


DIFFERENTIAL ROTATION
CONFIRMED BY
OBSERVATION OF DOUBLE SINE
WAVES



CO data here are older
Rotation curve remains
approx constant beyond R_0

- Very central part of Galaxy rotates like a rigid body, $\Theta \propto R$
- (Since $\Omega = \Theta/R$ is constant, all stars have same orbital period)
- For centrally concentrated mass, $\Theta \propto R^{-\frac{1}{2}}$ -Keplerian rotation
- No fall-off in Θ observed, suggesting substantial mass beyond R_0
- (note: most luminosity inside R_0)
- Similar rotation curves for other galaxies



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