

AY 20

Fall 2010

Rotation of the Milky Way

Reading: Carroll & Ostlie, Chapter 24.2, 24.3

Last class:

Introduced galactic coord system; defined Local Standard of Rest

Cylindrical coord system; origin at GC, R, θ, z

Corresponding velocity components: $\Pi \equiv dR/dt, \Theta \equiv d\theta/dt, Z \equiv dz/dt$

LSR is a point instantaneously centered on Sun and moving in a circular orbit along the solar circle about the Galactic Center

Velocity components: $\Pi_{\text{LSR}} \equiv 0, \Theta_{\text{LSR}} \equiv \Theta_0 = \Theta(R_0), Z_{\text{LSR}} \equiv 0$

Star's peculiar motion $u = \Pi - \Pi_{\text{LSR}}, v = \Theta - \Theta_{\text{LSR}} = \Theta - \Theta_0, w = Z - Z_{\text{LSR}}$

Velocity for any star relative to Sun: $\Delta u \equiv u - u_\odot, \Delta v \equiv v - v_\odot, \Delta w \equiv w - w_\odot$

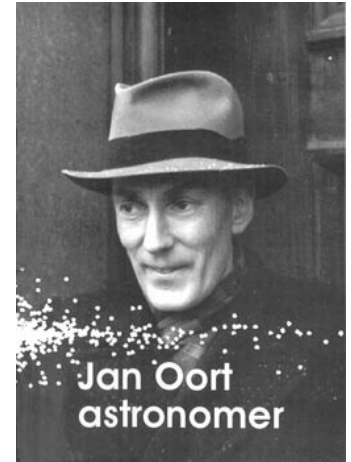
$$\therefore u_\odot = -\langle \Delta u \rangle, v_\odot = \langle v \rangle - \langle \Delta v \rangle, w_\odot = -\langle \Delta w \rangle$$

$$\text{Hence, } u_\odot = -10 \pm 0.4 \text{ km/s}$$

$$v_\odot = 5.2 \pm 0.6 \text{ km/s}$$

$$w_\odot = 7.2 \pm 0.4 \text{ km/s}$$

The Rotation of the Milky Way



- Flatness of MW suggests rotation about axis perpendicular to plane
- Observations of stars, gas confirm **differential** rotation
- i.e. Milky Way does not rotate like a rigid body
- angular velocity depends on distance from GC
- Observable effects of galactic rotation derived by Jan Oort
- Sun shares in differential rotation - has to be taken into account

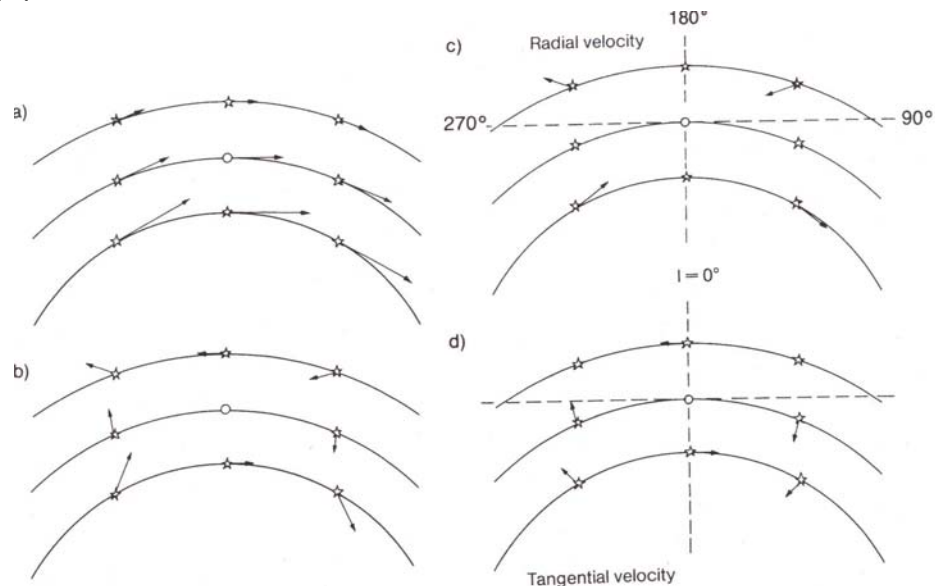
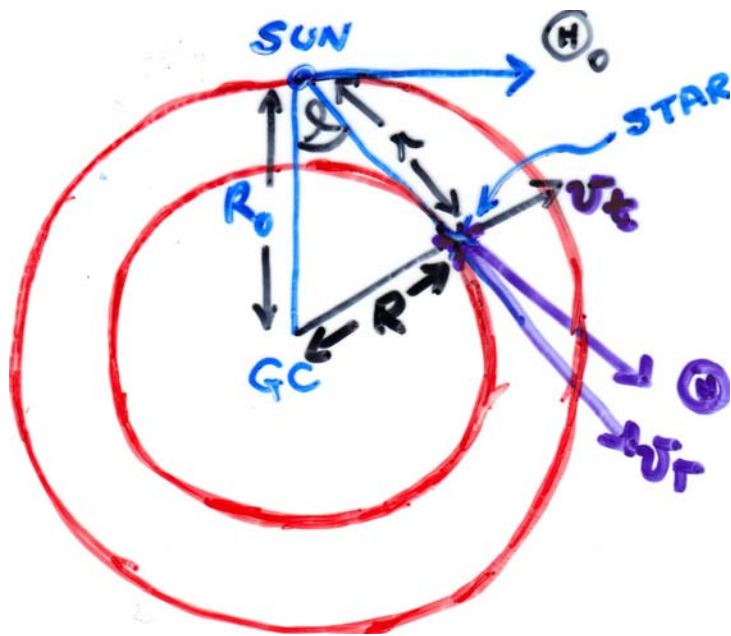


Fig. 18.13a–d. The effect of differential rotation on the radial velocities and proper motions of stars. (a) Near the Sun the orbital velocities of stars decrease outwards in the Galaxy. (b) The relative velocity with respect to the Sun is obtained by subtracting the solar velocity from the velocity vectors in (a). (c) The radial components of the velocities with respect to the Sun. This component vanishes for stars on the same orbit as the Sun. (d) The tangential components of the velocities



Implications of differential rotation: Oort's constants

Consider Sun and star orbiting GC
 star at R , with orbital velocity Θ
 Sun at R_0 from GC, orbital velocity Θ_0
 = orbital velocity of LSR since perfectly
 circular motion about GC assumed
 (Not unreasonable for Pop I stars and gas)

star at distance r from Sun, in dirⁿ of galactic longitude ℓ
 from Sun (i.e. observer's perspective),

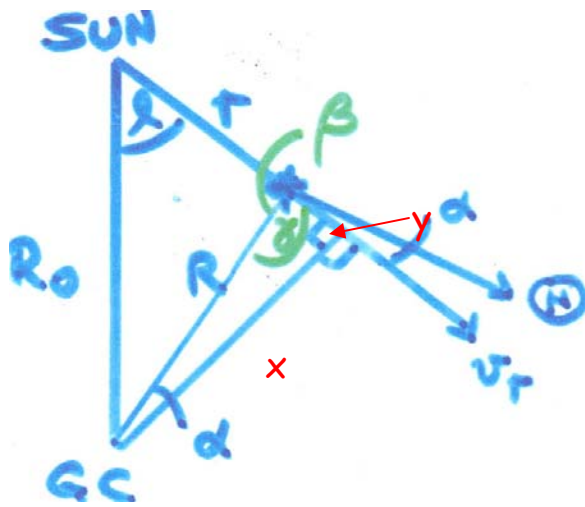
star has radial velocity, v_r , transverse velocity. v_t

$$\therefore v_r = \Theta \cos \alpha - \Theta_0 \sin \ell, \text{ and } v_t = \Theta \sin \alpha - \Theta_0 \cos \ell$$

$$\therefore \text{since angular velocity } \Omega(R) \equiv \Theta(R) / R$$

$$v_r = \Omega R \cos \alpha - \Omega_0 R_0 \sin \ell$$

$$v_t = \Omega R \sin \alpha - \Omega_0 R_0 \cos \ell$$



We have $v_r = \Omega R \cos \alpha - \Omega_0 R_0 \sin \ell$

and $v_t = \Omega R \sin \alpha - \Omega_0 R_0 \cos \ell$

$\sin \ell = x/R_0$ and $\cos \alpha = x/R$

$\therefore R_0/R = \cos \alpha / \sin \ell$ and $\rightarrow \cos \alpha = R_0 \sin \ell / R$

$\cos \ell = (r+y)/R_0 = (r + R \sin \alpha)/R_0$

$\therefore \sin \alpha = (R_0/R) \cos \ell - r/R$

$$v_r = \Omega R_0 \sin \ell - \Omega_0 R_0 \sin \ell \rightarrow v_r = R_0 (\Omega - \Omega_0) \sin \ell$$

$$\text{and } v_t = \Omega (R_0 \cos \ell - r) - \Omega_0 R_0 \cos \ell \rightarrow v_t = R_0 (\Omega - \Omega_0) \cos \ell - \Omega r$$

i.e. assuming circular symmetry, and R_0 , ℓ , and r known, can estimate $\Omega(R)$

BUT: r not known well unless nearby or variable, R_0 affected by extinction to GC

SO Oort assumed $\Omega(R)$ smoothly varying function of R

\therefore Taylor expansion of $\Omega(R)$ about $\Omega_0(R_0)$ is:

$$\Omega(R) = \Omega_0(R_0) + \left. \frac{d\Omega}{dR} \right|_{R_0} (R - R_0) + \dots$$

$$\Omega - \Omega_0 \approx \left. \frac{d\Omega}{dR} \right|_{R_0} (R - R_0) \quad \text{and } \Omega \approx \Omega_0$$

Taylor expansion valid only near Sun where $r \ll R_0$, and $R_0 \sim R$
 Since $\Omega \equiv \Theta/R$, can rewrite Taylor expansion:

$$\Omega - \Omega_0 = \frac{1}{R_0^2} \left[R_0 \left(\frac{d\Theta}{dR} \right) - \Theta_0 \right] (R - R_0)$$

Recall $v_r = R_0(\Omega - \Omega_0) \sin \ell$, $v_t = R_0(\Omega - \Omega_0) \cos \ell - \Omega r$

$$v_r = \left[\frac{d\Theta}{dR} \Big|_{R_0} - \frac{\Theta_0}{R_0} \right] (R - R_0) \sin \ell$$

$$v_t = \left[\frac{d\Theta}{dR} \Big|_{R_0} - \frac{\Theta_0}{R_0} \right] (R - R_0) \cos \ell - \Omega_0 r$$

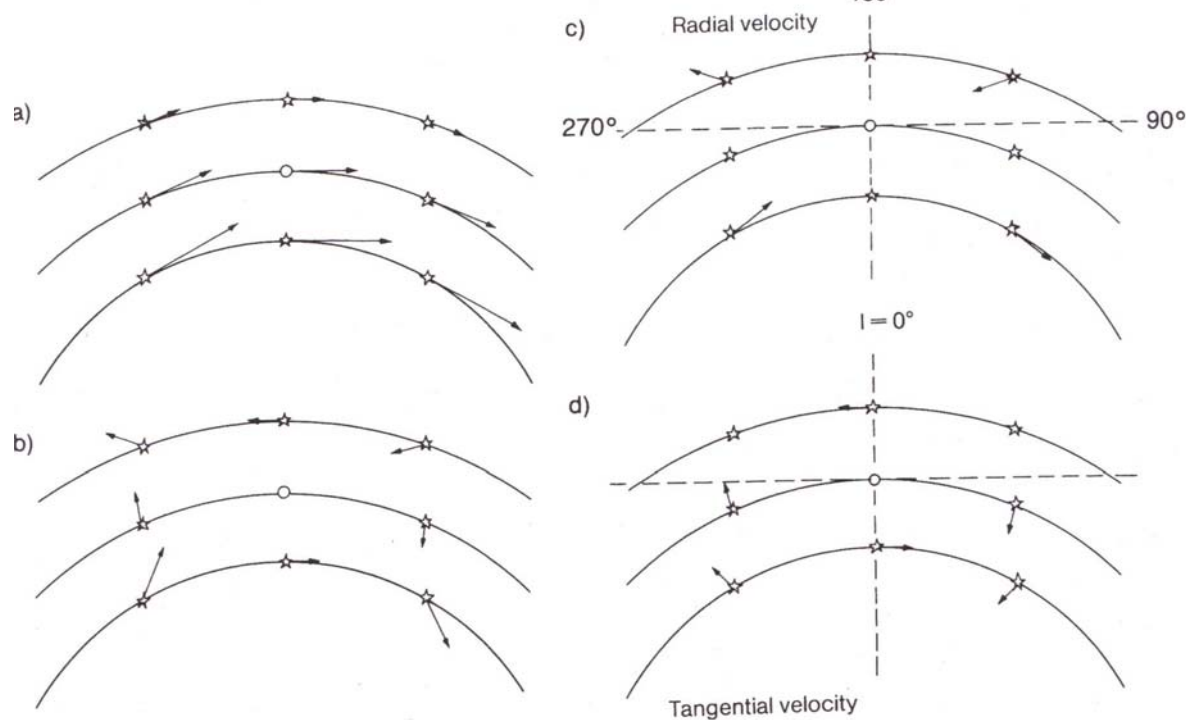
Define Oort's constants, A and B

$$A \equiv -\frac{1}{2} \left[\frac{d\Theta}{dR} \Big|_{R_0} - \frac{\Theta_0}{R_0} \right], B \equiv -\frac{1}{2} \left[\frac{d\Theta}{dR} \Big|_{R_0} + \frac{\Theta_0}{R_0} \right]$$

$$\therefore v_r \approx A \sin 2\ell$$

Explains shear rate

= rate of change of angular speed of rotation with distance from GC



Since $v_r \approx A \sin 2\ell$,
 observer (at Sun) sees different relative velocities

- For $\ell = 0^\circ$ or 180° ; $v_r = 0$, no motion in line of sight
- For $\ell = 90^\circ$ or 270° ; $v_r = 0$, motion on solar circle
- For $\ell = 45^\circ$; $v_r +ve$, star closer to GC, overtaking Sun
- For $\ell = 135^\circ$; $v_r -ve$, star lagging behind Sun

LIKEWISE EFFECT OF VORTICITY: (local rotation rate)

$$v_t = R_0 (\Omega - \Omega_0) \cos 2l - \Omega_0 \tau$$

$$R_0 - R = r \cos 2l$$

$$\Omega - \Omega_0 = 1/R_0^2 [R_0 (d\Omega/dR)_{R=R_0} - \Omega_0] (R - R_0)$$

$$\Rightarrow v_t = 1/R_0 [\Omega_0 - R_0 (d\Omega/dR)_{R=R_0}] r \cos^2 l - \Omega_0 \tau$$

$$\Omega_0 \tau \sim \Omega_0 \tau, \text{ NEAR SUN}$$

$$\text{AND } 2 \cos^2 l \sim 1 + \cos 2l$$

$$v_t = 1/2 [\Omega_0/R_0 - (d\Omega/dR)_{R=R_0}] r \cos 2l + 1/2 [\Omega_0/R_0 - (d\Omega/dR)_{R=R_0}] r - \Omega_0 \tau/R_0$$

$$\text{BUT } A = -1/2 [d\Omega/dR|_{R_0} - \Omega_0/R_0]$$

$$\text{AND } B = -1/2 [d\Omega/dR|_{R_0} + \Omega_0/R_0]$$

$$\therefore v_t = A \cos 2l + B r,$$

$$\therefore \text{and proper motion } \mu = v_t/r$$

$$\therefore \mu = A \cos 2l + B$$

$$v_r = A \sin 2l$$

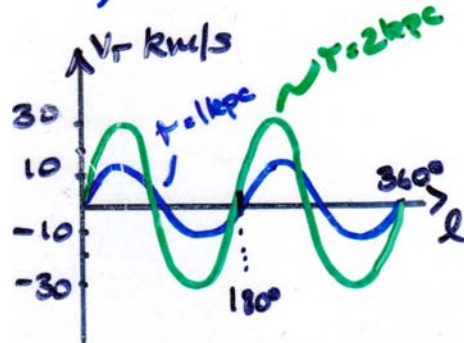
$$v_t = A \cos 2l + B$$

$$\mu = A \cos 2l + B$$

OBSERVED v_r FOR STARS AT SAME $l \rightarrow$ DOUBLE SINE CURVE WITH AMPLITUDE A

Hence A if r known

OBSERVED $\mu \rightarrow$ CURVE OFFSET $-B$



FOR STARS AT SAME DIST l
 v_r AS FUNCTION OF l

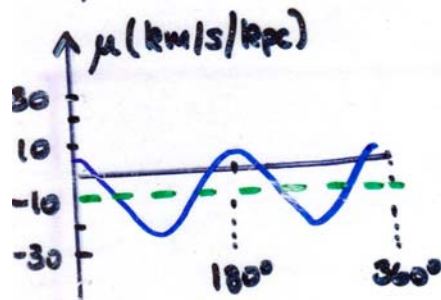
AMPLITUDE $\Rightarrow A$
 $= 15 \text{ km s}^{-1} \text{ kpc}^{-1}$
HIPPARCOS $\rightarrow 14.8 \pm 0.8$

μ INDEPENDENT OF DISTANCE

μ AS FUNCTION OF l

AMPLITUDE $\Rightarrow A$

"MEAN" $\Rightarrow B$
 $= -10 \text{ km s}^{-1} \text{ kpc}^{-1}$
HIPPARCOS $\rightarrow -12.4 \pm 0.6$



DIFFERENTIAL ROTATION
CONFIRMED BY
OBSERVATION OF DOUBLE SINE
WAVES

Implications: relating A, B to $R_0, \Theta_0, \Omega_0 = \Theta_0/R, d\Theta/dR|_{R_0}$

We have
$$A \equiv -\frac{1}{2} \left[\frac{d\Theta}{dR} \Big|_{R_0} - \frac{\Theta_0}{R_0} \right], B \equiv -\frac{1}{2} \left[\frac{d\Theta}{dR} \Big|_{R_0} + \frac{\Theta_0}{R_0} \right]$$

Thus $A - B = \Theta_0/R_0 = \Omega_0$ and $A+B = d\Theta/dR|_{R_0}$

$A = 14.8 \text{ km/sec/kpc}$ and $B = -12.4 \text{ km/sec/kpc}$

$\therefore \text{Rotation Period} = 2\pi/\Omega_0$

$= 2\pi \times 3 \times 10^{21} / 27.2 \times 10^5 \times 3 \times 10^7 \sim 2/9 \times 10^9 \text{ yrs} \sim 250 \text{ Myrs}$

\rightarrow Rotation period for **solar neighborhood** stars $\sim 250 \text{ Myrs}$

Angular velocity of LSR around GC $\sim 0.0053''/\text{year}$

But Taylor expansion not appropriate for studying **large scale** structure of Galaxy

Best probe = 21 cm HI line (not affected by I/S dust) - due to hyperfine structure in ground state of H atom

Can be several and separate clouds in line of sight

Use properties of differential rotation to determine distances

Due to differential rotation,
clouds on different orbits have
different velocities

$$v_r = R_0(\Omega - \Omega_0) \sin l$$

v_r increases until point where v_r
is maximum = $R_0(\Omega - \Omega_0)$

R_k is minimum distance from GC

$r = R_0 \cos l$ at that point

No HI measures of v_{\max} possible
for $90^\circ < l < 270^\circ$

Also, non-circular motions within
 20° of GC

Have to use other methods
e.g. Cepheids in plane

CO(1-0) rotational transition
(fewer CO clouds in line of sight)
etc,

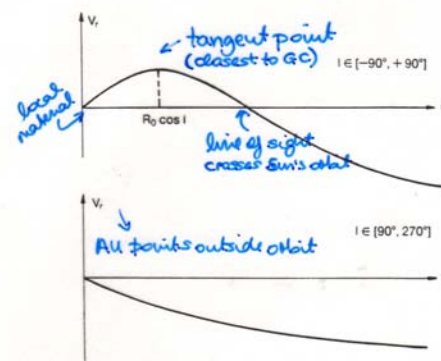
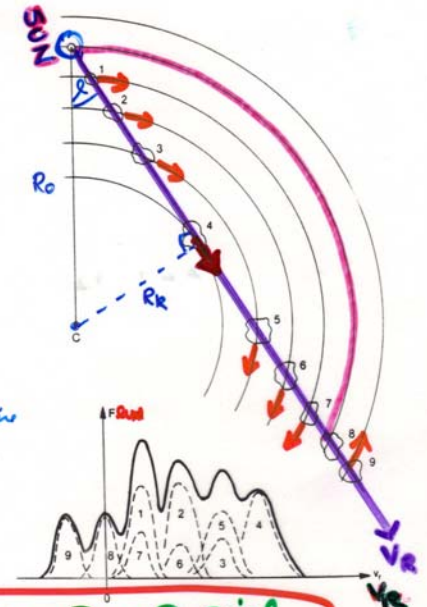


Fig. 18.17. The radial velocity as a function of distance (shown schematically) along a given line of sight

Fig. 18.18. Clouds at different distances have different velocities and therefore give rise to emission lines with different Doppler shifts. The observed flux density profile (continuous curve) is the sum of the line profiles of all the individual line profiles (dashed curves). The numbers of the line profiles correspond to the clouds in the upper picture

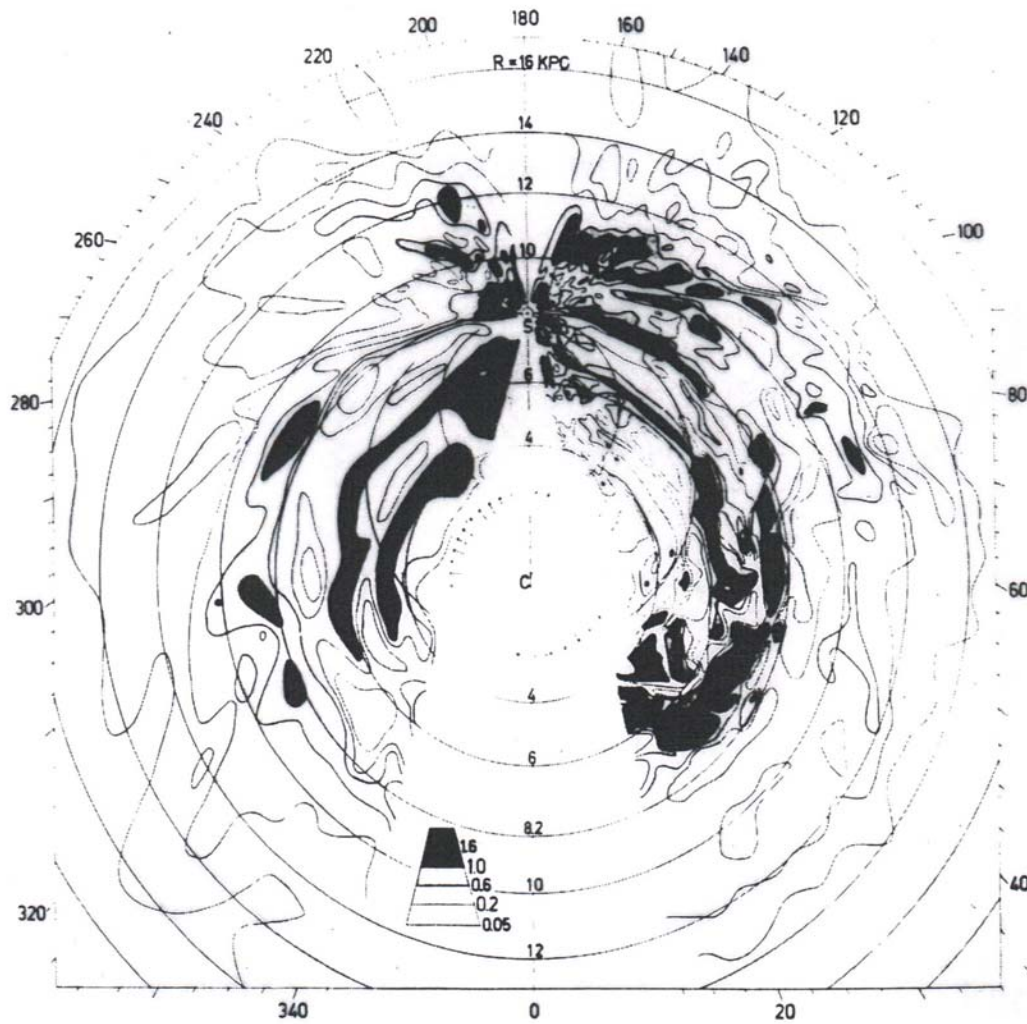


FOR TANGENT POINT $R_k = R_0 \sin l$
CAN MEASURE HIGHEST v_r AT
DIFFERENT l 's *

GREATEST VELOCITY AT ④ WHERE LINE OF SIGHT
IS TANGENTIAL TO RADIUS

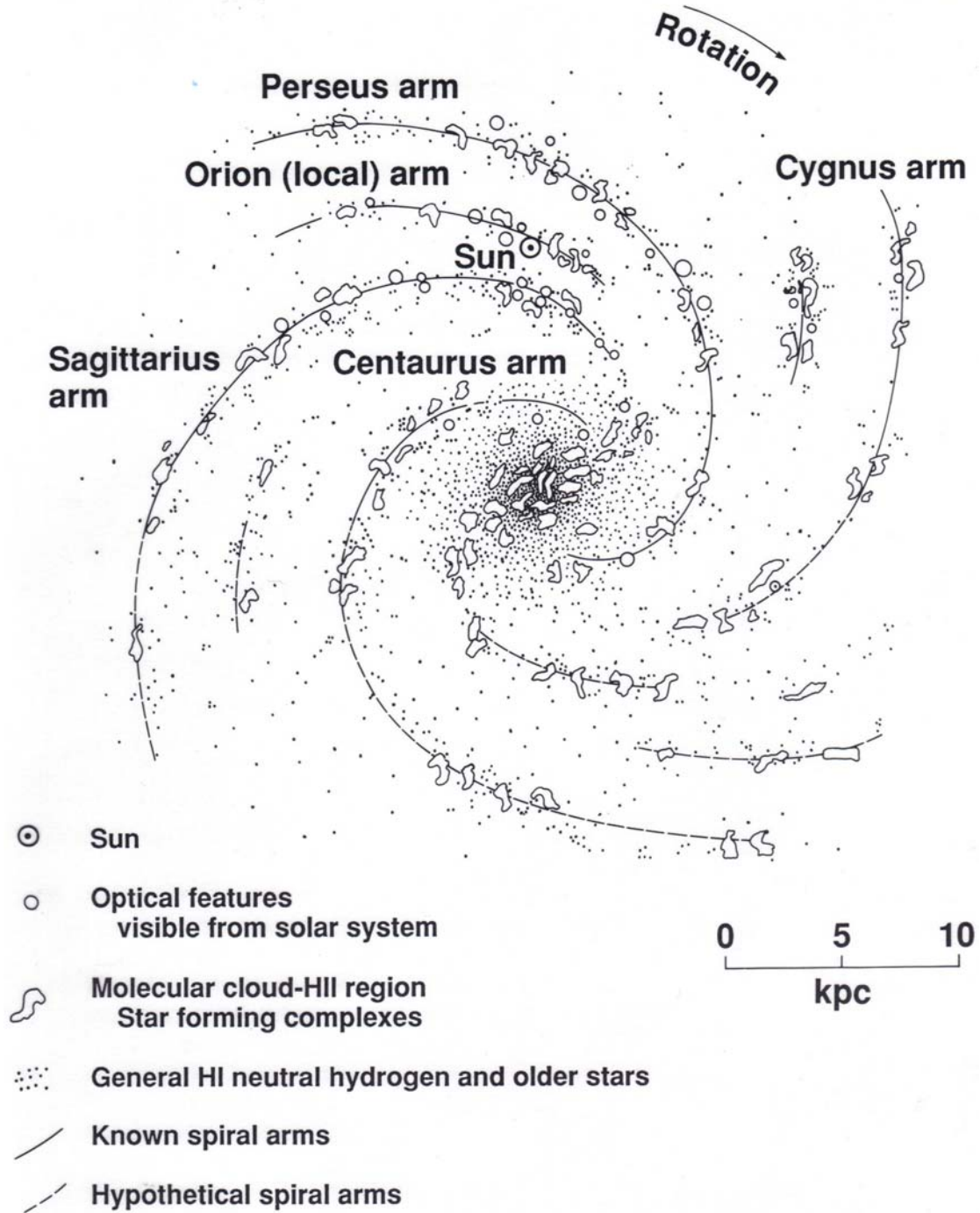
→ CLOUD ④ MOVES FASTEST AWAY
CLOUD 8 UN-MOVING
CLOUD 9 METHOD FAILS } RELATIVE TO SUN

* → ROTATION CURVE OF GALAXY
WITHIN SOLAR GALACTOCENTRIC RADIUS



Results of 21cm mapping
suggest spiral structure
But significant gaps

FIG. 7.4. Radio map at 21 cm wavelength of the distribution of neutral hydrogen based on combined observations at Leiden (Holland) and Sydney (Australia). Distances from the centre are marked in kiloparsecs (1 kpc = 3260 light years). (After Kerr and Westerhout, 1964)



Rotation Curve for Milky Way Galaxy also obtained
= plot of rotation speed as function of galactocentric radius

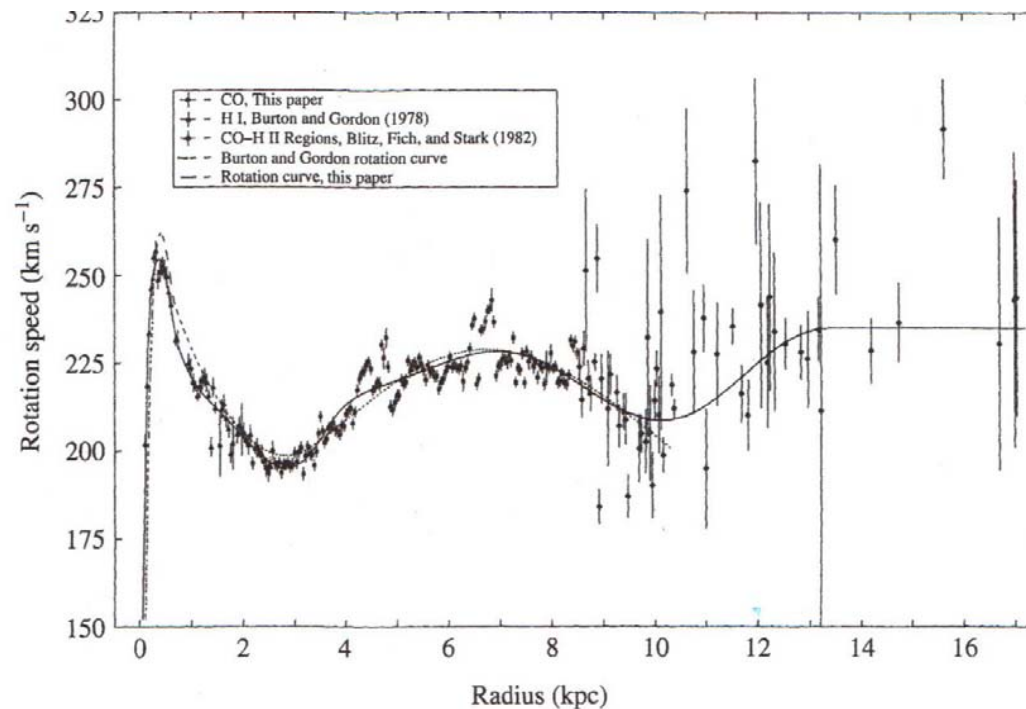
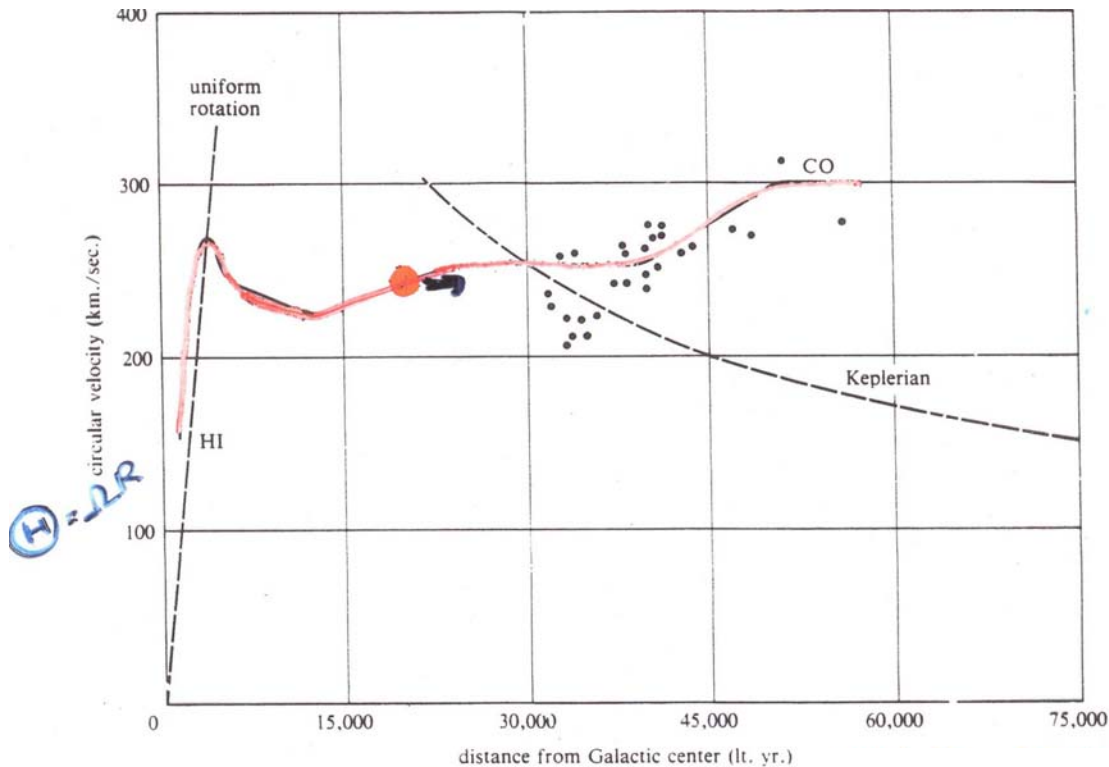
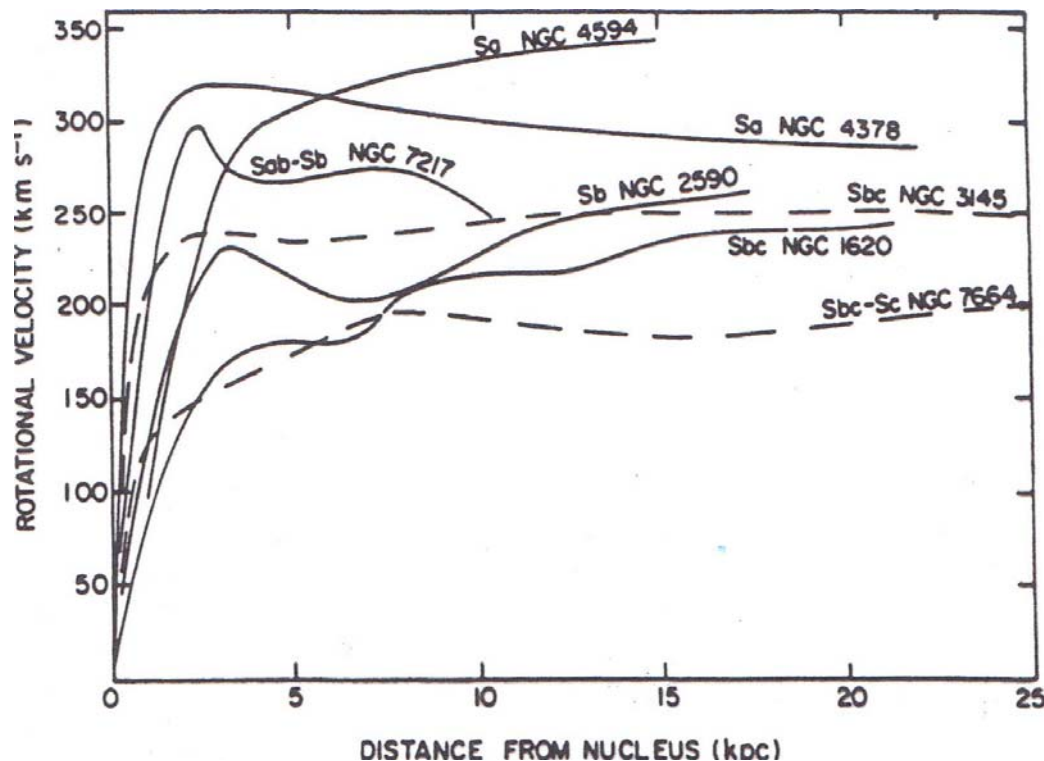


FIGURE 24.25 The rotation curve of the Milky Way Galaxy. The 1985 IAU standard values of $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km s⁻¹ have been assumed. (Figure adapted from Clemens, *Ap. J.*, 295, 422, 1985.)



CO data here are older
Rotation curve remains
approx constant beyond R_0

- Very central part of Galaxy rotates like a rigid body, $\Theta \propto R$
- (Since $\Omega = \Theta/R$ is constant, all stars have same orbital period)
- For centrally concentrated mass, $\Theta \propto R^{-\frac{1}{2}}$ -Keplerian rotation
- No fall-off in Θ observed, suggesting substantial mass beyond R_0
- (note: most luminosity inside R_0)
- Similar rotation curves for other galaxies



Flat rotation curves extending to large radii imply unexpected matter density distributions

Recall: force on a star of mass m due to a mass M_r interior to star's position is given by $mV^2/r = GM_r m/r^2$ (spherical symm holds)

$$\therefore M_r = V^2 r / G \text{ and } dM_r/dr = V^2 / G$$

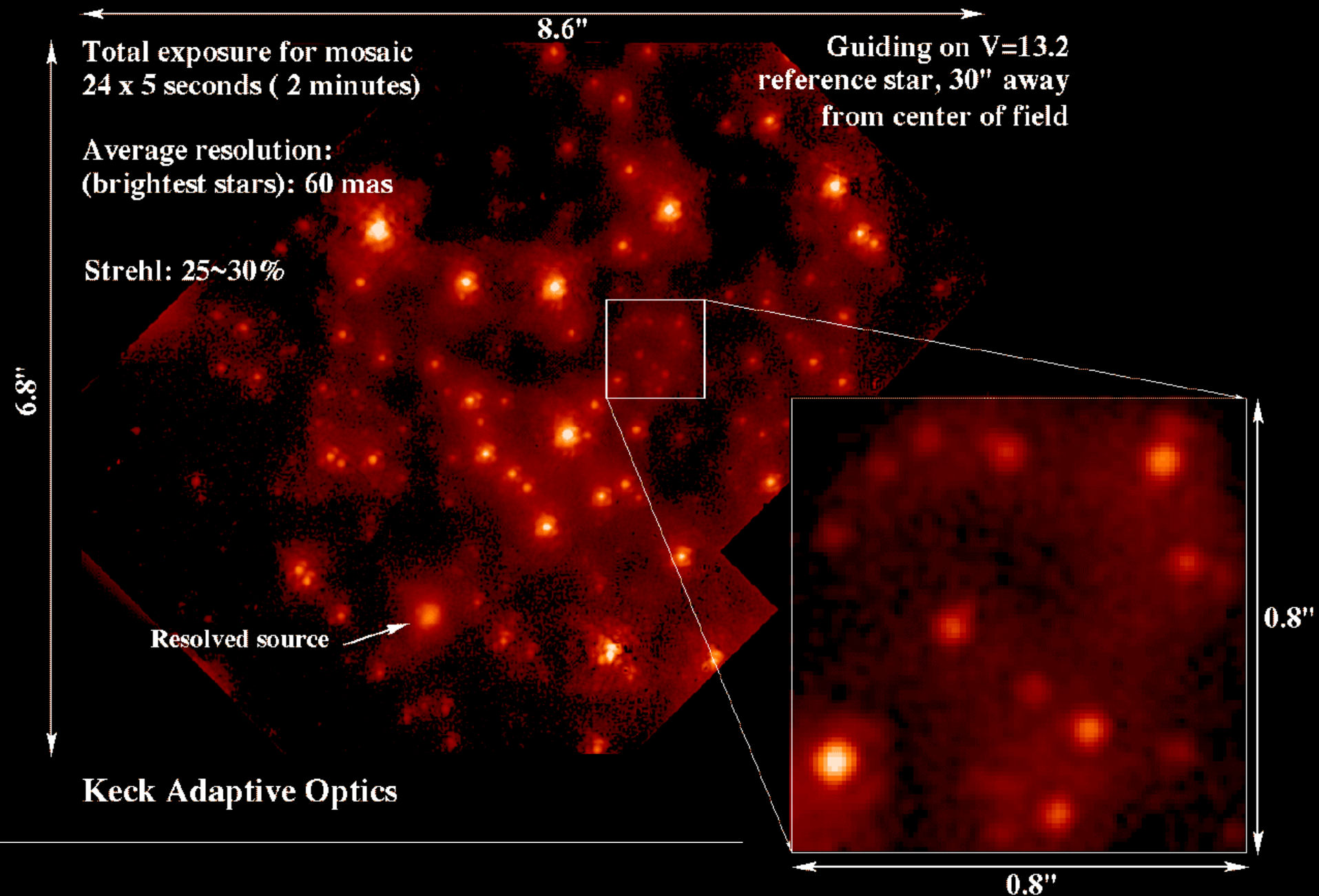
Mass conservation equation: $dM_r/dr = 4\pi r^2 \rho$

$$\therefore \rho(r) = V^2 / 4\pi r^2 G \rightarrow \rho(r) \propto r^{-2}$$

But density of stars in halo $\propto r^{-3.5}$

Rapid drop off of stellar density suggests most of galaxy mass is non-luminous dark matter

The Galactic Center at 2.2 microns (with adaptive optics)



At the Galactic Center

Exotic radio source Sgr A*

From radio maser spots, size < 100 AU

$A_v > 30$ but K, M giants detected at $2.2\mu\text{m}$ (Andrea Ghez et al. UCLA)

Motions of stars indicate central mass:

e.g. star SO-2, $P = 15.2$ yrs, $e = 0.87$,
perigalact distance = 120 AU

semi-major axis $a_{S2} = r_{\text{perihelion}}/(1-e)$
 $\sim 1.4 \times 10^{14}$ m

$M = 4\pi^2 a_{S2}^3 / GP^2 \sim 3.7 \times 10^6 M_{\odot}$
(Kepler's 3rd)

SUPER MASSIVE BLACK HOLE

