AY 20 Fall 2010

Interstellar Gas
Formation of Stars

Reading: Carroll & Ostlie, Chapter 12.1, 12.3

From last class

In ISM, dust mass ~ 1/100 gas mass

Nevertheless, extinction mainly due to dust

= many non-spherical particles

graphite or silicate cores with icy mantles, PAHs

sizes ~ few x 0.1 μm $\rightarrow$ Å scales (PAHS)

ISM principally hydrogen, HI, H_2, HII

Diffuse HI clouds detected from 21 cm line observations

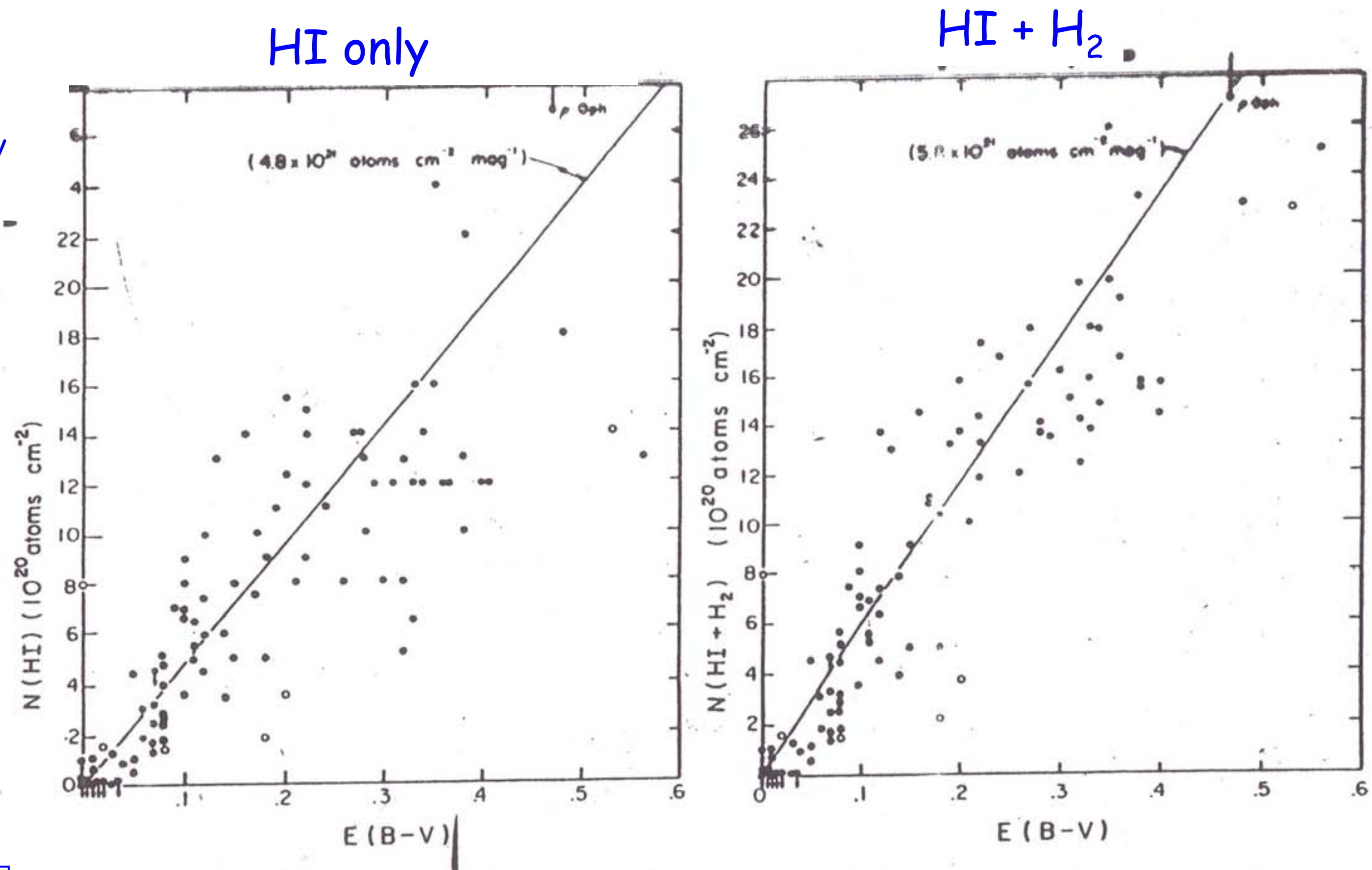
Absorption at 21cm due to spin-flip transition in H atom

$\tau_{HI} \propto N_{HI}$ in optically thin case and $A_V \sim \tau_{dust} \propto N_{dust}$

Plots of N_{HI} versus A_V show good correlation $N_{HI,}$ & N_{dust}

$\rightarrow$ gas and dust well mixed for $A_V < 3$ $\rightarrow$ galaxy structure

Column density of hydrogen correlates with A_v (or E_{B-V})



N_{dust} from UV spectra

Figure 2 Correlations between gas column densities and interstellar reddening for 100 stars from the *Copernicus* atomic and molecular hydrogen survey (Savage et al. 1977, Bohlin, Savage & Drake 1978): (*a*) shows the atomic hydrogen column density, N(HI), versus $E(B-V)$, (*b*) shows the total hydrogen column density, $N(HI+H_2) = N(HI) + 2N(H_2)$, versus $E(B-V)$. Be stars are denoted with the open symbols. The solid line in (*a*) gives the average *atomic* hydrogen to $E(B-V)$ ratio 4.8×10^{21} atoms cm^{-2} mag^{-1}. In (*b*) the solid line gives the average *total* hydrogen to $E(B-V)$ ratio of 5.8×10^{21} atoms cm^{-2} mag^{-1}. The point for ρ Oph in (*a*) and (*b*) should be moved upward by about a factor of 2.7.

Savage & Mathis 1979, ARAA 17, 86

UV radiation (λ < 912 Å) from O and B stars ionizes HI

Central star temperatures required > 30,000K

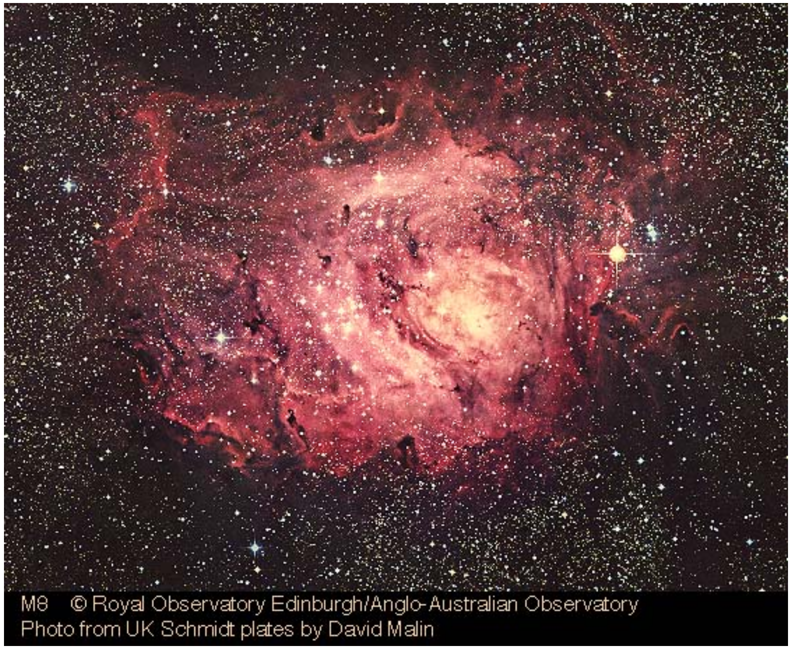
Recombination lines when protons, free electrons recombine to excited state

photons of different energies emitted in cascade to ground state

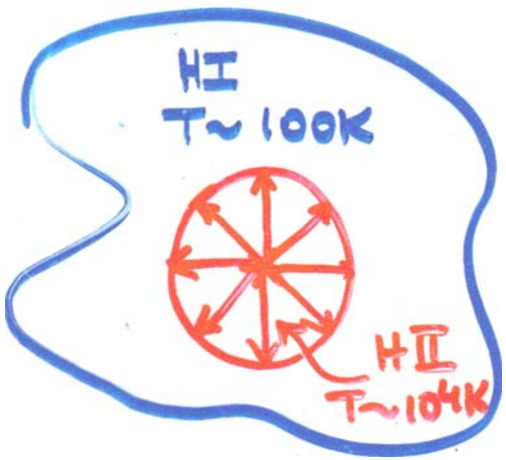
n = 3$\rightarrow$2 transition dominates i.e. Hα (6565 Å)

$\therefore$emission from HII regions is red

Lagoon nebula in Sagittarius



Size of HII regions



Massive star embedded in dense H_2 cloud
stellar UV photons dissociate H_2, ionize HI
simultaneously, electrons, protons recombine
→ more HI

Each ionization removes a photon from beam
& star has fixed possible UV photon output
∴ size of region ionized is limited

For uniform density, ionization spreads isotropically, filling a sphere
→ Strömgren sphere

Require that ionization balance holds at each location within sphere

i.e. ionization rate for any parcel of gas =
recombination rate of protons and electrons in same parcel

Also, ionization rate = rate at which ionizing photons emitted by star
= N_* (recall: ionizing photons have energy > 13.6 eV; i.e. λ < 912Å)

For stars with spectral types O4 to B2, masses 70 to 10 $M_\odot$, values of log N_* vary from 49.9 to 44.8/sec

Determining the Strömgren radius

Ionization balance requirement:

Number of UV photons/unit time/unit volume =
Number of recombinations /unit time /unit volume

And number of recombinations /unit time /unit volume $\propto$ number density of electrons x number density of atoms

$= \alpha n_e n_p$ (where α is recombination coefficient = probability of recombining)

$$\therefore N_* / 4/3\pi r^3 = \alpha n_e n_p = \alpha n_e^2$$

$$\therefore r^3 = 3N_*/4\pi\alpha \times n_e^{-2/3}$$

$\therefore$ Strömgren radius, $R_S = \left(\frac{3N_*}{4\pi\alpha}\right)^{\frac{1}{3}} n_e^{-2/3}$

VERY DRAMATIC BOUNDARY!

CONSIDER AN O6 STAR, $T_{eff} \sim 45{,}000K$, $L = 1.3 \times 10^5 L_\odot$

$$\therefore \lambda_{max} = 0.29/T_{eff} = \frac{29 \times 10^{-2} \times 10^8}{45 \times 10^3} A^\circ$$

$$= 3/5 \times 10^3 \text{Å} \sim 600 \text{Å} \quad (660\text{Å})$$

LOTS OF $\lambda < 912$Å PHOTONS → IONIZATION

AT 660 Å, EACH PHOTON HAS ENERGY $E_\gamma = hc/\lambda$

$$= \frac{6.6 \times 10^{-27} \times 3 \times 10^{10}}{660 \times 10^{-8} \times 1.6 \times 10^{-12}} \sim 20\,\text{ev}$$

LET E_γ = AVERAGE ENERGY/PHOTON

& NUMBER OF IONIZING PHOTONS = $N = L/E_\gamma$

$$\therefore N = \frac{1.3 \times 10^5 \times 4 \times 10^{33}\,\text{ergs}}{20 \times 1.6 \times 10^{-12}} \sim 1.6 \times 10^{49}$$

Need to evaluate → SIZE OF STRÖMGREN SPHERE, R_s

$$= \left(\frac{3N}{4\pi\alpha}\right)^{1/3} n_H^{-2/3}$$

For HII region at 8000K

$\alpha = 3.1 \times 10^{-13}\,\text{cm}^3\,\text{s}^{-1}$ AND $n_e \sim 100\,\text{cm}^{-3}$

$$\therefore R_s = \left(\frac{3 \times 1.6 \times 10^{49}}{4\pi \times 3.1 \times 10^{-13}}\right)^{1/3} (100)^{-2/3} \cdot \frac{1}{3 \times 10^{18}}\,\text{pc}$$

$$\sim \left(\frac{160}{4\pi \times 27}\right)^{1/3} \times \left(\frac{10^{60}}{10^{18}}\right)^{1/3} \times 10^2$$

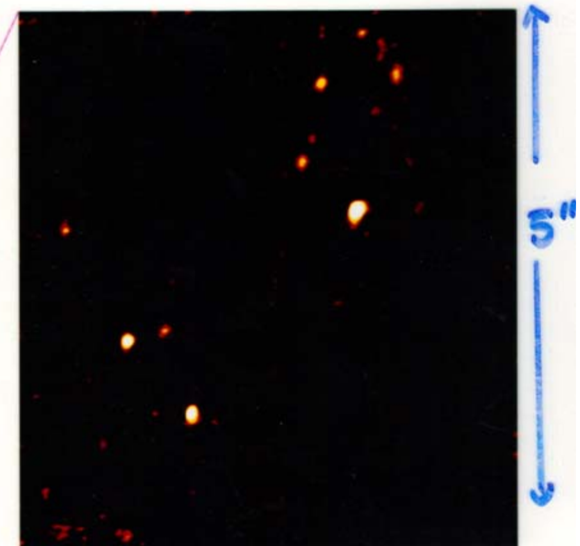
$\therefore R_s = 3.5\,\text{pc}$ $= 3.5 \times 3.1 \times 10^{18} / 1{,}5 \times 10^{13}$ AU

Stars form in molecular clouds (H_2)

A Nursery for Stars

This optical photograph from the Palomar Oschin telescope shows many young stars clustered around an opaque black area in the constellation of Serpens. Such dark areas are usually not empty sky. Instead, they represent dense dust condensations where new stars are likely to form. The large quantities of dust prevent the light emitted by any embedded stars from reaching us.

By contrast, an image of the same region, taken by NASA's Infrared Astronomical Satellite (IRAS) reveals the presence of a strong source of infrared radiation. Optical radiation from newly-born stars is absorbed by the surrounding dust particles but re-radiated by them at infrared wavelengths. The intense infrared emission from the Serpens core is a signpost that a number of young stars are present.



The Owens Valley millimeter array image provides a much more detailed picture of star formation activity in the very heart of the Serpens infrared region. The image above shows the emission that dust particles emit at a wavelength of 3 mm. Each bright region indicates the presence of a dense dust clump that is either about to form a star, prestellar, or already harbors a star nucleus, protostellar. We can count at least 26 such clumps. Most exciting from an astronomer's point of view - the number of clumps per given mass conforms to the same pattern as that expected for isolated stars. We are viewing the fragmentation of a single core to create a cluster of infant stars:

an interstellar nursery!

STILL A MAJOR PROBLEM FOR 21ST CENTURY

HOW DO STARS AND PLANETS FORM?

NEED TO CONNECT THEORY AND OBSERVATION

THEORY:

STARS ARE BORN FROM GRAVITATIONAL CONTRACTION OF INTERSTELLAR CLOUDS

CONTRACTION → GRAVITATIONAL ENERGY

↙ THERMAL ENERGY ↘ RADIATIVE ENERGY

⇒ HEATING TO POINT WHERE FUSION CAN OCCUR → NUCLEAR ENERGY

⇓

PROTO-STAR

OBSERVATIONS:

STELLAR BIRTH MOSTLY OBSCURED IN HIGH A_V, MOLECULAR CLOUDS

OPTICAL: INFERENCES FROM SCATTERED LIGHT

INFRARED & MILLIMETER WAVELENGTHS: INFERENCES FROM DUST PROPERTIES

DIRECT OBSERVATIONS

DUST, GAS - SOMETIMES NEAR IR STARS

WHAT ARE CONDITIONS FOR CORE COLLAPSE?

FROM VIRIAL THEOREM $2K + U = 0$
FOR A STABLE, GRAVITATIONALLY BOUND SYSTEM
($K \equiv$ INTERNAL KINETIC ENERGY, U = GRAVITATIONAL POTENTIAL ENERGY)

IF $2K > |U|$, GAS PRESSURE DOMINATES → EXPANSION

IF $2K < |U|$ GRAVITATIONAL FORCE DOMINATES → COLLAPSE

RECALL $U = -\frac{3}{5}\frac{GM_c^2}{R_c}$ where M_c = cloud mass, R_c = cloud radius & cloud spherical at constant ρ_o

$K = \frac{3}{2} NkT = \frac{3}{2}\frac{M_c}{\mu m_H} kT$ μ = mean molecular weight, N = number of particles

∴ FOR COLLAPSE $\frac{3M_c}{\mu m_H} kT < \frac{3}{5}\frac{GM_c^2}{R_c}$

for CONSTANT density ρ_o, $R_c = \left(\frac{3M_c}{4\pi\rho_o}\right)^{1/3}$

∴ $\frac{kT}{\mu m_H} < \frac{GM_c}{5}\left(\frac{4\pi\rho_o}{3M_c}\right)^{1/3}$ & $\frac{5kT}{G\mu m_H} < \left(\frac{4\pi\rho_o}{3}\right)^{1/3} M_c^{2/3}$

∴ FOR COLLAPSE, M_c MUST BE AT LEAST SOME MINIMUM MASS M_J

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_o}\right)^{1/2}$$ = Jeans Mass

Jeans Criterion: for collapse $M_c > M_J$

Jeans Criterion also applies to cloud size

FOR COLLAPSE $R_c > R_J$, JEANS LENGTH

SINCE $R_c = \left(\frac{3M_c}{4\pi\rho_0}\right)^{1/3}$

$$R_J = \left(\frac{3}{4\pi\rho_0}\right)^{1/3} M_J^{1/3}, \text{ where } M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

$$= \left(\frac{3}{4\pi\rho_0}\right)^{1/3}\left(\frac{5kT}{G\mu m_H}\right)^{1/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/6}$$

$$= \frac{3^{1/2}\,5^{1/2}(kT)^{1/2}}{(4\pi\rho_0 G\mu m_H)^{1/2}}$$

$\therefore$ JEANS LENGTH $R_J = \left(\frac{15kT}{4\pi G\mu m_H \rho_0}\right)^{1/2}$

$\therefore$ EASIER FOR COOLER, DENSER CORES TO COLLAPSE : $R_J \propto \left(\frac{T}{\rho_0}\right)^{1/2}$

HERE, EXTERNAL PRESSURE NEGLECTED. WHEN INCLUDED, CRITICAL MASS = BONNER-EBERT MASS

$$M_{BE} = \frac{C_{BE}\, v_T^4}{P_0^{1/2} G^{3/2}}, \quad v_T = \text{isothermal sound speed} \sim \sqrt{\frac{P}{\rho}} = \sqrt{\frac{kT}{\mu m_H}}$$

$C_{BE} \simeq 1.18$

$C_J = 5.46$

EXAMPLES

$$M_J = \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$$

$M_\odot \sim 2\times10^{33}$ gms : $\rho_0 = n_H m_H$ (n_H = # atoms/cm^3)

$\Rightarrow \quad \rho_0 = 1.7\times10^{-24}\, n_H$

$$M_J = \left(\frac{5\times1.38\times10^{-16}}{6.67\times10^{-8}\times1\times1.67\times10^{-24}}\right)^{3/2}\left(\frac{3}{4\pi}\right)^{1/2} \times \frac{1}{2\times10^{33}} \times \frac{1}{(1.7\times10^{-24})^{1/2}}\sqrt{\frac{T^3}{n_H}}$$

$$\therefore M_J \simeq \left(\frac{1}{3}\right)^{3/2} 10^{24}\left(\frac{3}{4\pi}\right)^{1/2}\frac{1}{10^{21}}\times\frac{1}{3} = 30\sqrt{\frac{T^3}{n_H}}\, M_\odot$$

FOR A DIFFUSE HI CLOUD: $T = 100K$, $n_H = 20$

$$\therefore M_J = 30\times\left(\frac{10^6}{20}\right)^{1/2} = 15\times\left(\frac{10^6}{5}\right)^{1/2} \sim 7\times10^3 M_\odot$$

BUT $M_{HI\,CLOUD}$ USUALLY $< 7\times10^3 M_\odot$

$\therefore$ no collapse

FOR A MOLECULAR CORE: $T = 100K$ $n_H \sim 10^8$

$$\therefore M_J = 30\times\left(\frac{10^6}{10^8}\right)^{1/2} \sim 3 M_\odot$$

typical core mass $\sim 10-100 M_\odot \Rightarrow$ collapse

COLLAPSE TIME-SCALE

- IF $M_J < M_{CLOUD} \rightarrow$ COLLAPSE
- IF CLOUD OPTICALLY THIN, GRAVITATIONAL ENERGY RADIATED AWAY $\rightarrow$ NO TEMPERATURE CHANGE
- PRESSURE GRADIENT TOO SMALL TO PREVENT/RESIST COLLAPSE : $dP/dr \ll \frac{GM_r\rho}{r^2}$

$\therefore$ ISOTHERMAL, FREE-FALL COLLAPSE

RECALL DERIVATION OF EQUN. HYDROSTATIC EQUIL^{M}

$$\rho \frac{d^2r}{dt^2} = -\frac{GM_r}{r^2}\rho - \frac{dP}{dr} \left[= 0 \text{ IN STATIC CASE}\right]$$

HERE $\frac{d^2r}{dt^2} = -\frac{GM_r}{r^2}$ AND $M_r = \frac{4}{3}\pi r_0^3 \rho_0$

M_r = MASS INTERIOR TO r = ENCLOSED MASS AT ONSET OF COLLAPSE

$$\therefore \frac{dr}{dt}\cdot\frac{d^2r}{dt^2} = -\frac{4\pi G}{3}\rho_0 r_0^3 \cdot \frac{1}{r^2}\cdot\frac{dr}{dt} \Rightarrow \frac{1}{2}\left(\frac{dr}{dt}\right)^2 = +\frac{4\pi G}{3}\rho_0\frac{r_0^3}{r} + C_1$$

At $r = r_0$, $\frac{dr}{dt} = 0$, $\therefore C_1 = -\frac{4\pi G}{3}\rho_0 r_0^2$

$\therefore$ VELOCITY "AT SURFACE, r," OF COLLAPSING "SPHERE"

$$\frac{dr}{dt} = -\left[\frac{8\pi G}{3}\rho_0 r_0^2\left(\frac{r_0}{r} - 1\right)\right]^{1/2}$$

($-$) –VE GRADIENT BECAUSE COLLAPSE

CARROLL & OSTLIE P. 415 →

TO INTEGRATE $\frac{dr}{dt} = -\left[\frac{8\pi}{3} G \rho_0 r_0^2 \left(\frac{r_0}{r} - 1\right)\right]^{1/2}$

SET $\theta = r/r_0$ $\quad K = \left(\frac{8\pi G}{3}\rho_0\right)^{1/2}$

$\therefore d\theta/dt = -K\left(\frac{1}{\theta} - 1\right)^{1/2}$

SET $\theta \equiv \cos^2\xi \quad \Rightarrow \quad \cos^2\xi \frac{d\xi}{dt} = -\frac{K}{2}$

$\therefore \xi/2 + 1/4 \sin 2\xi = \frac{K}{2}t + C_2$

WHEN $t = 0$, $r = r_0$, $\theta = 1$, $\xi = 0$

$\therefore C_2 = 0$

$\therefore$ GRAVITATIONAL COLLAPSE OF CLOUD GIVEN BY:

$$\xi/2 + 1/2 \sin 2\xi = Kt$$

FREE-FALL TIME = TIME WHEN $r = 0$

i.e. $\theta = 0$ AND $\xi = \pi/2$

$\therefore t_{ff} = \pi/2K = \left[3\pi/32G\rho_0\right]^{1/2}$

DYNAMICAL TIME-SCALE = FREE FALL TIME

$$= t_{ff} = \left[\frac{3\pi}{32}\frac{1}{G\rho_0}\right]^{1/2}$$

$\therefore t_{ff}$ INDEPENDENT OF RADIUS OF INITIAL SPHERE. FOR UNIFORM DENSITY, ρ_0, ALL PARTS OF CLOUD TAKE SAME TIME TO COLLAPSE

$\equiv$ HOMOLOGOUS COLLAPSE

NOTE: HOMOLOGOUS COLLAPSE $t_{ff} \propto [1/G\rho_0]^{1/2}$

ALL PARTS OF CLOUD COLLAPSE AT SAME RATE (NO RADIAL DEPENDENCE)

DENSITY INCREASES AT SAME RATE THROUGHOUT

<u>BUT</u> INITIALLY DENSITY <u>MUST</u> BE UNIFORM

IF COLLAPSING CORE DENSITY NON-UNIFORM

e.g. SOMEWHAT CENTRALLY CONDENSED

$\rightarrow$ INSIDE-OUT COLLAPSE

TYPICAL TIME-SCALES FOR HOMOLOGOUS COLLAPSE:

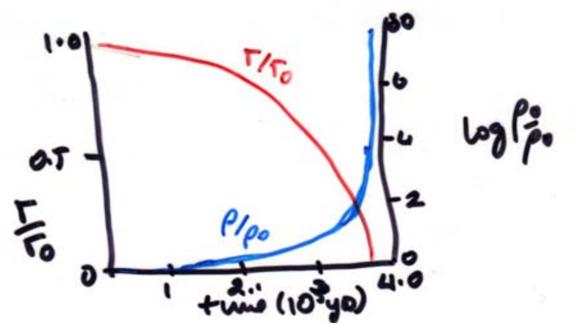
ASSUME $T = 10K$ $n_{H_2} \sim 10^{10} m^{-3}$, $\rho_0 = 2 m_H n_{H_2}$
$= 3 \times 10^{-17} kg\, m^{-3}$

For n_{H_2}, $\mu = 2$

$$\therefore t_{ff} = \left[\frac{3\pi}{32}\frac{1}{G\rho_0}\right]^{1/2} = \left[\frac{3\pi}{32}\frac{1}{6.7\times10^{-11}\times 3\times10^{-17}}\right]^{1/2}$$

$$= 10^{13}\left(\frac{3\pi}{6.7}\right)^{1/2} \sim \frac{3}{2.5}\times 10^{13} secs = \frac{3\times10^{13}}{3\times 2.5\times 10^{7}} yrs$$

$$\therefore t_{ff} \approx 4\times10^{5} yrs$$



Numerical solution for homologous collapse

LOWER LIMIT TO MASS OF FRAGMENTS?

TOTAL GRAVITATIONAL POTENTIAL ENERGY

$$U_g = -\frac{3}{5}\frac{GM^2}{R}$$

& ENERGY RADIATED AWAY DURING COLLAPSE $= \frac{1}{2} U_g$

∴ LUMINOSITY DUE TO GRAVIT^L CONTRACTION

$$= L_{ff} \simeq \frac{3}{10}\frac{GM^2}{R}\cdot\frac{1}{t_{ff}} = \frac{3}{10}\frac{GM^2}{R}\cdot\left(\frac{32G\rho_0}{3\pi}\right)^{1/2}$$

& $\rho_0 = \frac{3M}{4\pi R^3}$ $\Rightarrow \frac{3}{10}\frac{GM^2}{R}\left(\frac{32GM\times 3}{12\pi^2 R^3}\right)^{1/2}$

$$\therefore L_{ff} \sim G^{3/2}\frac{M^{5/2}}{R^{5/2}}$$

not quite thermodynamic equilibrium

FOR OPTICALLY THICK CLOUD $L_{RAD} = 4\pi R^2 \sigma T^4 \times e$

ADIABATIC $e \sim 0$

THERMODYNAMIC EQUILIBRIUM $e \sim 1$

IF $L_{RAD} = L_{ff}$, $4\pi R^2 \sigma T^4 e = G^{3/2}\frac{M^{5/2}}{R^{5/2}}$

$$\therefore M^{5/2} = \frac{4\pi}{G^{3/2}} R^{9/2} e \sigma T^4 .$$

↖ EXPRESS IN TERMS OF ρ_0, M_J,

⇒ MINIMUM JEANS MASS

⇒ FRAGMENTATION CEASES AT $\sim 0.5 M_\odot$

(ASSUME ADIABATIC EFFECTS BEGIN $\sim T = 1000K$, AND $\mu \sim 1$, $e \sim 0.1$)

STAR FORMATION: CLOUD COLLAPSE

1. MOLECULAR CLOUD $M \sim 1000 M_\odot$ $D \sim 10$pc (diameter)

2. CLOUD CONTRACTS - ALL HEAT RADIATED
   DENSITY INCREASES, M_J DECREASES

ISOTHERMAL COLLAPSE — RECALL $M_J \propto (T^3/\rho)^{1/2}$

3. PROBABLY INITIAL DENSITY INHOMOGENITIES

∴ SECTIONS OF CLOUD INDEPENDENTLY HAVE $M > M_J$ LOCALLY
i.e. FOR LOWER R_J
⇒ LOCAL COLLAPSE CENTERS

[FRAGMENTATION ⇒ MULTIPLE STARS CAN FORM]

? WHEN DOES FRAGMENTATION STOP?

4. DENSITY INCREASES IN EACH FRAGMENT UNTIL OPTICALLY THICK - NO ENERGY (RAD^N) THEN ESCAPES ≡ ADIABATIC COLLAPSE

5. SINCE NO ESCAPE OF RADIATION TEMPERATURE BEGINS TO INCREASE
   ∴ FOR SAME ρ, M_J INCREASES

RECALL $M_J \simeq \left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_0}\right)^{1/2}$

FOR ADIABATIC COLLAPSE $T \propto \rho^{\gamma-1}$

$\therefore M_J \propto \rho^{(3\gamma-3)/2}/\rho^{+1/2}$ $\therefore M_J \propto \rho^{(3\gamma-4)/2}$

FOR HYDROGEN $\gamma = 5/3$ $\rightarrow M_J \propto \rho^{1/2}$

pre-main sequence evolution and planetary system formation