AY 20 Fall 2010

Stellar Interiors
The SUN

Reading: Carroll & Ostlie, Chapter §10.6, Chapter 11

Stellar structure equations, constitutive relations, plus boundary conditions:
$\rightarrow$Vogt & Russell theorem
stellar composition & mass uniquely determine radius, luminosity, internal structure

$$\text{recall } \frac{dP}{dr} P = -\frac{GM_r\rho}{r^2}, \therefore P_c = \frac{GM_*\rho}{R_*}$$

$$\text{and P} = \frac{\rho kT}{\mu m_H}, \therefore T_c = \frac{P_c \mu m_H}{\rho k}$$

Changes in composition (X, Y, Z) due to nuclear burning $\rightarrow$ changes to μ

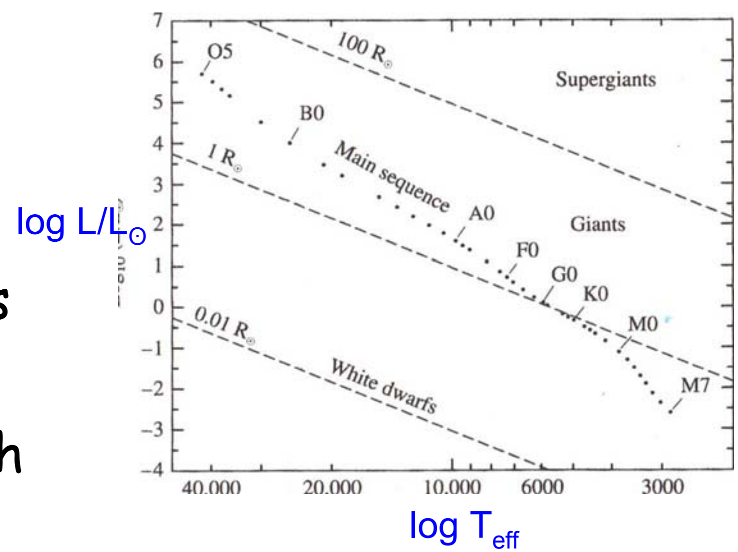
pp chain, CNO cycle are slow $\rightarrow$
slow composition changes; most stars have similar composition

$\therefore$ Smooth change of structure with mass

As M increases, P_c,T_c increase*

$\rightarrow$ pp chain in low mass stars, CNO in high

From theoretical models $\rightarrow$ HR diagram



H-burning stars lie on main sequence

Main sequence $\equiv$ mass sequence

Implications for main sequence stars

$$5 \times 10^{-4} L_\odot < L_* < 1 \times 10^6 L_\odot$$

$$0.08\ M_\odot < M_* < 90\ M_\odot$$

$$2700\ K < T_{eff} < 53{,}000\ K$$

radiation from higher mass stars >> from lower mass stars

→ reserves of fuel used faster; lifetimes of higher mass stars shorter

spectra change with T_{eff} and hence with M_*

→ spectral type (O B A F G K M) appropriate abscissa for H-R diagram

convection zones at different levels:

Upper m-s: CNO cycle; high T dependence; rapid change of ε with r
→ convection in H-burning core, radiation outside

As M_* decreases: T_c decreases; pp chain dominates; core becomes radiative

At surface, T_{eff} decreases, κ increases → convection

"Surface convection zone" increases in depth with decreasing mass; at 0.3 $M_\odot$ stars are fully convective

Eddington Limit: at very high T, radiation pressure dominates P.
M and L limited by hydrostatic equilibrium condition

$$L_{ed} / L_\odot \approx 3.8 \times 10^4\ M/M_\odot$$

For main sequence stars: theoretical $M_* < 100 M_\odot$; observed limit ~ $70 M_\odot$

SUN - nearest example of a star

The Sun is a typical main sequence star. Its principal properties are:

mass	m	$= 1.989 \times 10^{30}$ kg	$\sim 2 \times 10^{33}$ gm
radius	R	$= 6.960 \times 10^{8}$ m	$\sim 7 \times 10^{10}$ cm
mean density		$= 1409$ kg m^{-3}	
central density	ρ_c	$= 1.6 \times 10^{5}$ kg m^{-3}	
luminosity	L	$= 3.9 \times 10^{26}$ W	$\sim 4 \times 10^{33}$ ergs/cm²/sec
effective temperature	T_e	$= 5785$ K	
central temperature	T_c	$= 1.5 \times 10^{7}$ K	
absolute bolometric magnitude	M_{bol}	$= 4.72$	
absolute visual magnitude	M_v	$= 4.79$	
spectral class		G2 V	
colour indices	B-V	$= 0.62$	
	U-B	$= 0.10$	
surface chemical composition	X	$= 0.71$	MASS FRACTIONS
	Y	$= 0.27$	
	Z	$= 0.02$	
rotational period			
at the equator		25 d	
at 60° latitude		29 d	

T_e and L → G2V star on main sequence

Composition has changed over 4.57×10^9 years but not much at surface.

99% of solar energy from pp chain within $\frac{1}{4}R_\odot$

Model of P, T, M, energy production as function of r based on above properties,

From observations

Direct measures → solar constant (energy/unit/time/unit area)

$$F_\odot = 1.36 \times 10^6 \text{ ergs/sec/cm}^2$$

$$\therefore L = 4\pi d^2 F = 4\pi \times (1AU)^2 F = 4\pi \times 2.25 \times 10^{26} \times 1.36 \times 10^6$$

$$L_\odot \sim 3.9 \times 10^{33} \text{ erg/sec}$$

Sun's diameter at Earth ~ 0.5° = 32′

$$R_\odot = 1.5 \times 10^{13} \tan\theta = 6.96 \times 10^{10} \text{ cm}$$

For a blackbody $L_\odot = \sigma T^4_{eff} 4\pi R^2_\odot$

$$\therefore T^4_{eff} = 4 \times 10^{33}/5.67 \times 10^{-5} \times 4\pi \times 6.96^2 \times 10^{20}$$

$$= 10^{18}/5.7 \times \pi \times 5 \times 10 \sim 10^{16}$$

$$T_{eff} = 5800 \text{ K}$$

Mass from one body (earth) orbiting another (sun)

$$M_\odot = 2 \times 10^{33} \text{ gms, and } \langle\rho_\odot\rangle = 1.4 \text{ gm/cm2}$$

ONE BODY ORBITING ANOTHER → MASS DETERMINATION

FOR EARTH: ORBITAL RADIUS = 1 AU

PERIOD = 3.16×10^7 sec

$$\therefore V = \frac{2\pi d}{P} = \frac{2\pi \times 1.5 \times 10^{13}}{3 \times 10^7}$$

$$\sim 3 \times 10^6 \text{ cm s}^{-1}$$ (67,000 mph)

$$\frac{G M_\odot m_{EARTH}}{d^2} \sim \frac{mv^2}{d}$$

$$\therefore M_\odot = \frac{dv^2}{G} = \frac{1.5 \times 10^{13} \times 9 \times 10^{12}}{6.7 \times 10^{-8}}$$

$$\sim \frac{15}{7} \times 10^{33} \text{ gms}$$

$$\therefore M_\odot \sim 2 \times 10^{33} \text{ gms}$$

$$\text{MEAN DENSITY} = \bar{\rho} = \frac{M_\odot}{4/3 \pi R_\odot^3} = \frac{2 \times 10^{33}}{4/3 \pi (7)^3 \times 10^{30}}$$

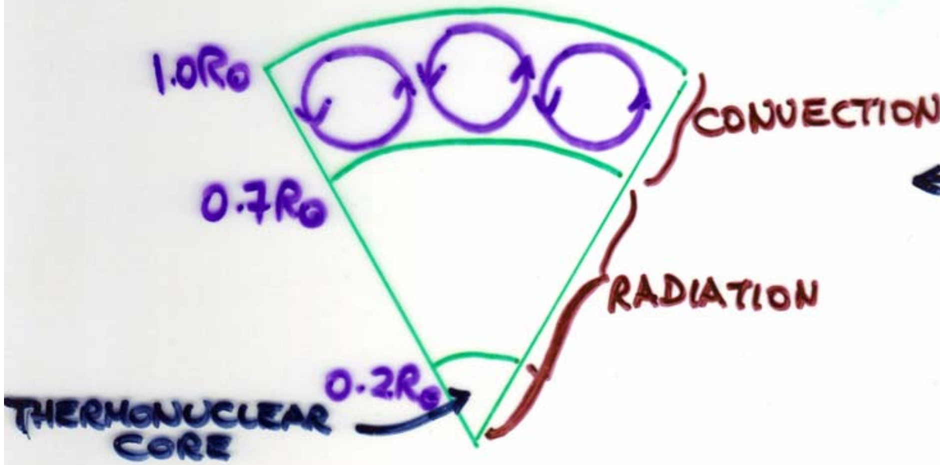
$$\therefore \bar{\rho} \sim \frac{5 \times 10^2}{50 \times 7} \sim 1.4 \text{ gm cm}^{-3}$$

$\bar{\rho}_\odot = 1.4$ gm cm^{-3}, $\bar{\rho}_{EARTH} = 5.5$ gm cm^{-3}

LIQUID WATER 1 gm cm^{-3}

KNOW MASS & COMPOSITION

∴ CONSTRUCT STANDARD MODEL



← SCHEMATIC OF STANDARD MODEL

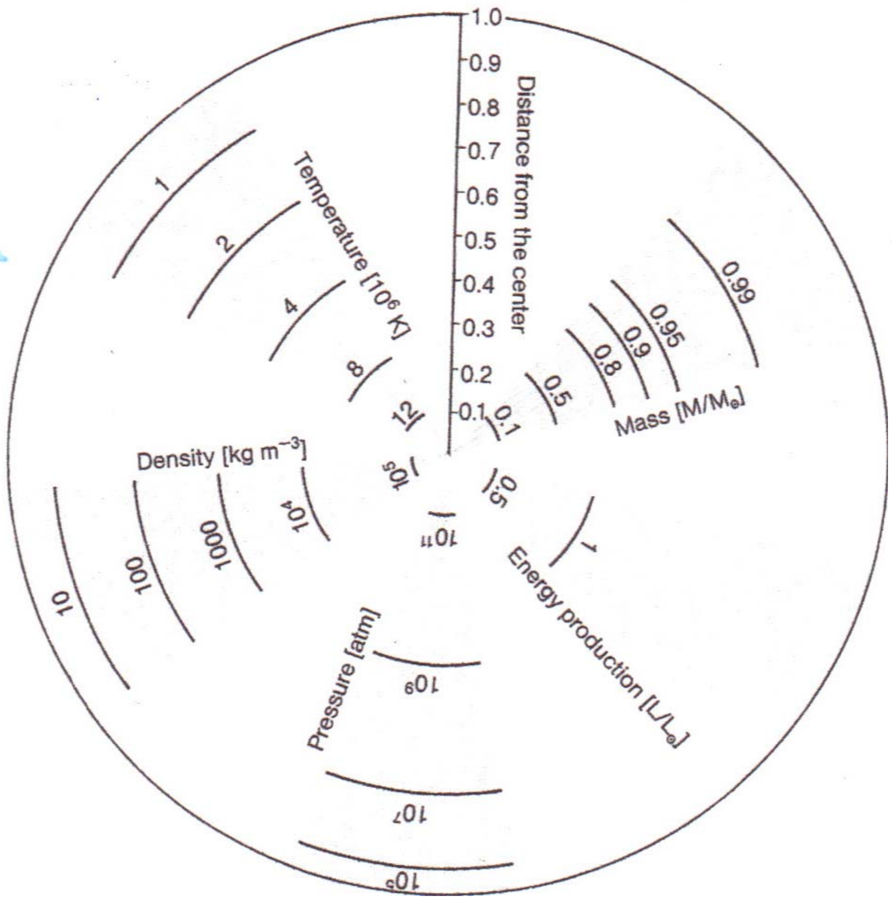
ONE MODEL: DURING SUN'S LIFETIME

IN CORE { X HAS DECREASED 0.71 → 0.34
Y " INCREASED 0.27 → 0.64

AT SURFACE X INCREASE TO 0.73

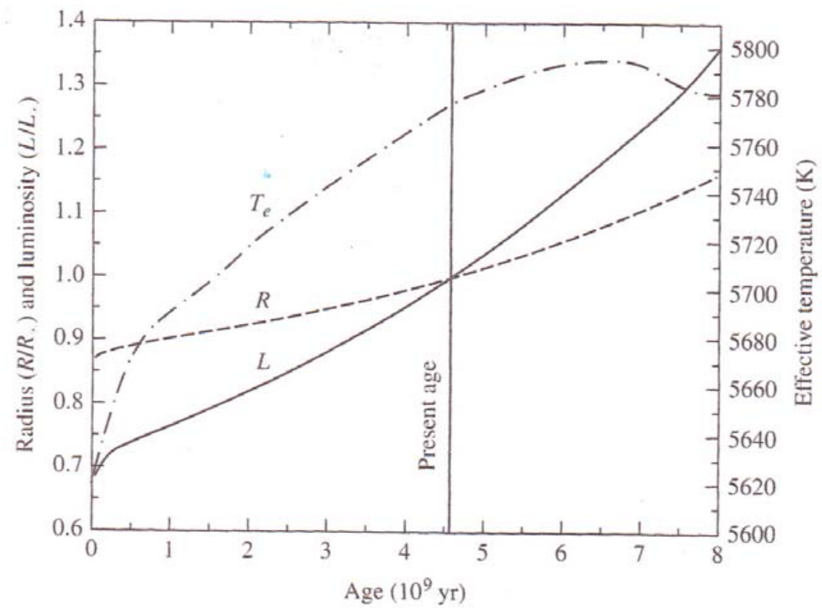
COMPOSITION CHANGES → CHANGES TO L_*, R_*, T_{eff}

Sun: Model



Composition changes → evolution

from Bahcall et al 2001



Original composition modified due to
Nuclear burning
Surface convection
Settling of metals
As mass fractions change, μi, μn change
T, L, R change

MODELS OF SUN RELY ON SAME STRUCTURE EQUATIONS, BOUNDARY CONDITIONS, ITERATIVE INTEGRATION TECHNIQUES AS FOR STARS

IN TERMS OF COMPOSITON

REGION	RADIAL SIZE	COMPOSITION
CORE	$0.2 R_\odot$	He H metals 0.63 0.35 0.02 (almost ionized)
RADIATIVE ZONE	$0.5 R_\odot$	0.23 0.75 0.02 (highly ionized)
CONVECTIVE ZONE	$0.3 R_\odot$	X, Y, Z same less ionized
PHOTOSPHERE	$0.002 R_\odot$	"
SOLAR SURFACE	$1.000 R_\odot$	—
CHROMOSPHERE	$0.02 R_\odot$	"
CORONA	$\simeq 5$	same, highly ionized

N.B. VISIBLE SURFACE = PHOTOSPHERE
- 300 - 500 km thick
AND T = 4500-8000 K

$\frac{dT}{dr}$ fulfills convection conditions between surface and $\sim 0.7 R_\odot$

(1) $^1_1H + ^1_1H \rightarrow ^2_1H + e^+ + \nu_e$

(2) $^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$

(3) $^3_2He + ^3_2He \rightarrow ^4_2He + 2\,^1_1H$

COMPOSITION STRUCTURE FOR H, He, ^3_2He

At top of zone burning H, T is lower

⇒ REACTIONS (1),(2) MORE LIKELY THAN (3)

⇒ MORE ^3_2He

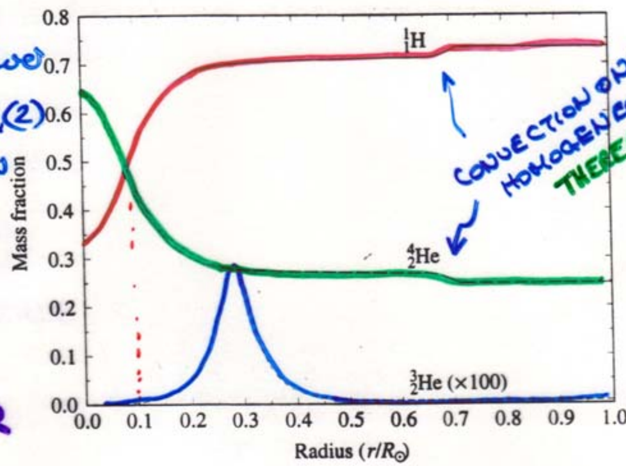


Figure 11.3 The abundances of $^1_1$H, $^3_2$He, and $^4_2$He as a function of radius for the Sun. (Data from ~~Guzik, private communication, 1994.~~) Bahcall et al 2001

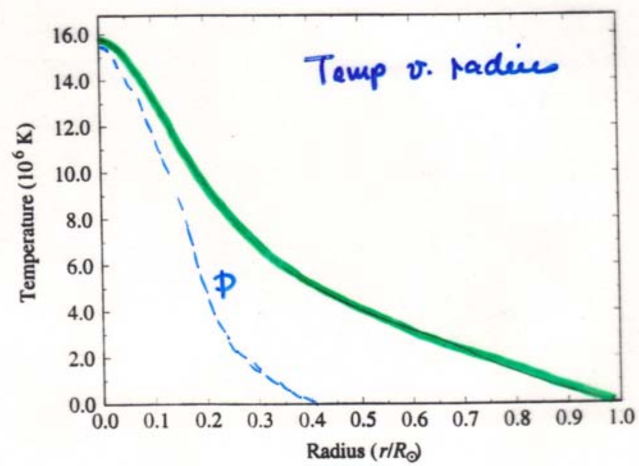


Figure 11.4 The temperature in the solar interior as a function of radius. (Data from ~~Guzik, private communication, 1994.~~) Bahcall et al 2001

LARGEST CONTRIBUTION TO SOLAR ENERGY FROM $r = 0.1 R_\odot$

RECALL MASS CONSERVATION EQUN:

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

$$\therefore dM_r = 4\pi r^2 \rho \, dr = \rho \, dV$$

FOR SPHERICAL SHELL

∴ MASS IN GIVEN INTERVAL INCREASES WITH r AND ρ

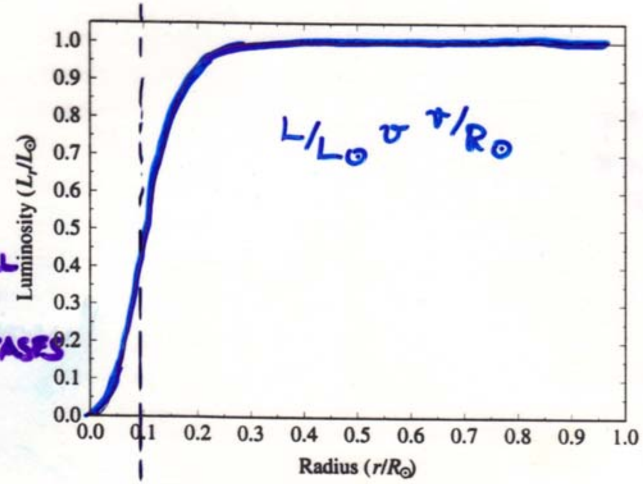
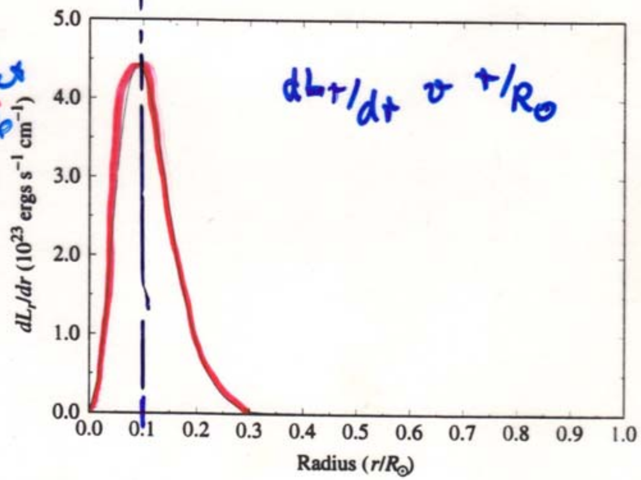


Figure 11.5 The interior luminosity of the Sun as a function of radius. (~~Data from Guzik, private communication, 1994.~~) Bahcall 2001)

∴ energy emitted will peak when significant amount of mass available to produce energy (not at center)



COMPARE W.

ρ v $r/R_\odot$

L v $r/R_\odot$

Figure 11.6 The derivative of the Sun's interior luminosity with respect to radius, showing the location of the greatest contribution to the energy output. (Data from ~~Guzik, private communication, 1994.~~) Bahcall 2001)

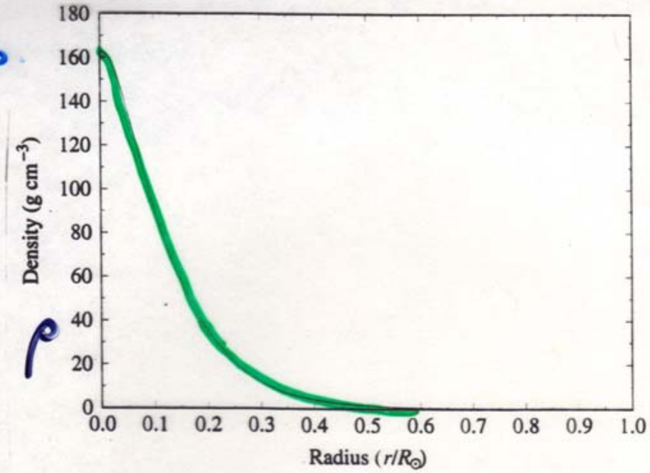
INTERIOR PRESSURE, DENSITY FALL OFF RAPIDLY

RECALL

$$\frac{dP}{dr} = -\frac{GM_r \rho}{r^2}$$

(HYDROSTATIC EQUIL^{M})

$$P = \frac{\rho k T}{\mu m_H}$$

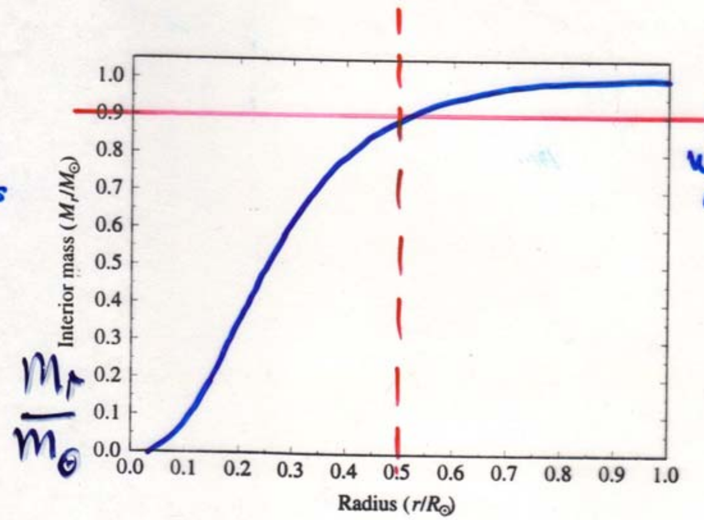


DIRECT RESULT OF REQUIRING HYDROSTATIC EQUILIBRIUM, IDEAL GAS LAW, AND COMPOSITION STRUCTURE

ALSO SIMILAR PRESSURE FALL OFF

igure 11.7 The density structure of the Sun as a function of radius.)ata from ~~Guzik~~, private communication, ~~1994~~.)

* DUE TO RAPID RISE IN ρ TOWARDS CENTER

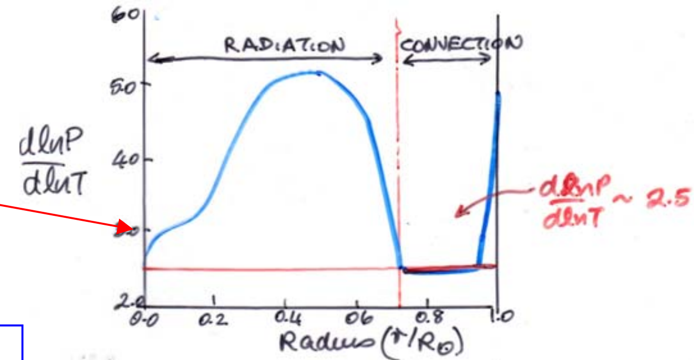


90% * WITHIN $0.5 R_\odot$

Figure 11.9 The interior mass as a function of radius for the Sun. (Data from Guzik, private communication, 1994.)

How is energy transported out from interior?

In deep core, approaching convectiive transport



Model of Sun reproduces M. L, R, T, & surface compositions pretty well
-also fits evolutionary timescales

BUT lithium abundance anomalously low
→refine applications of convection, rotation, mass-loss

RECALL CONVECTION CRITERION

$$\left|\frac{dT}{dr}\right|_{actual} > \left|\frac{dT}{dr}\right|_{adiabatic}$$

ie TEMP GRADIENT IS SUPERADIABATIC

→ CONDITION FOR ONSET OF CONVECTION

$$\frac{d\ln P}{d\ln T} < 2.5$$

∴ ONSET OF CONVECTION AT $r = 0.714 R_\odot$

IN OUTER REGIONS OPACITY HIGH, PREVENTS RADIATIVE TRANSPORT

ABOVE $r = 0.95 R_\odot$ INVOKE MIXING LENGTH

TESTING THE STANDARD MODEL

MODEL PREDICTS NEUTRINO PRODUCTION

SHOULD DETECT ν_e – LOW INTERACTION X-SECTION

∴ SHOULD NOT BE IMPEDED

CHLORINE ISOTOPE $^{37}_{17}Cl$ CAN INTERACT WITH ν_e's TO PRODUCE $^{37}_{18}Ar$ (half-life 35 days)

$$^{37}_{17}Cl + \nu_e \rightleftharpoons {}^{37}_{18}Ar + e^-$$

threshold energy 0.814 MeV

∴ 615,000 kg of cleaning fluid C_2Cl_4 tetrachlorethylene in 100,000 gallons in Homestead Mine in South Dakota – Ray Davis

Most neutrinos from PP III chain

$$^{8}_{5}B \rightarrow {}^{8}_{4}Be + e^+ + \nu$$ (not frequent!)

TANK PURGED EVERY FEW MONTHS, ARGON ATOMS COUNTED

CAPTURE RATE IN TERMS OF SNU – solar neutrino unit

1 SNU = 10^{-36} reactions / target atom / sec

BAHCALL MODELS PREDICTED 7.9 SNU from 1970-94

DAVIS FOUND 2.56 ± 0.16

SUPER KAMIOKANDE, GALLEX AT GRAN SASSO OTHER EXPERIMENTS

→ SIMILAR DISCREPANCIES

⇒ COMPLETE & INTENSE STUDY OF THE PHYSICAL PRINCIPLES IN SOLAR MODEL, ε, κ, X, Y, Z etc. etc.

MSW Effect can explain
MIKHEYEV – SMIRNOV – WOLFENSTEIN

→ NEUTRINOS TRANSFORMED AS PASS THROUGH SUN (electroweak theory of particle physics)

pp chain → ν_e electron neutrinos

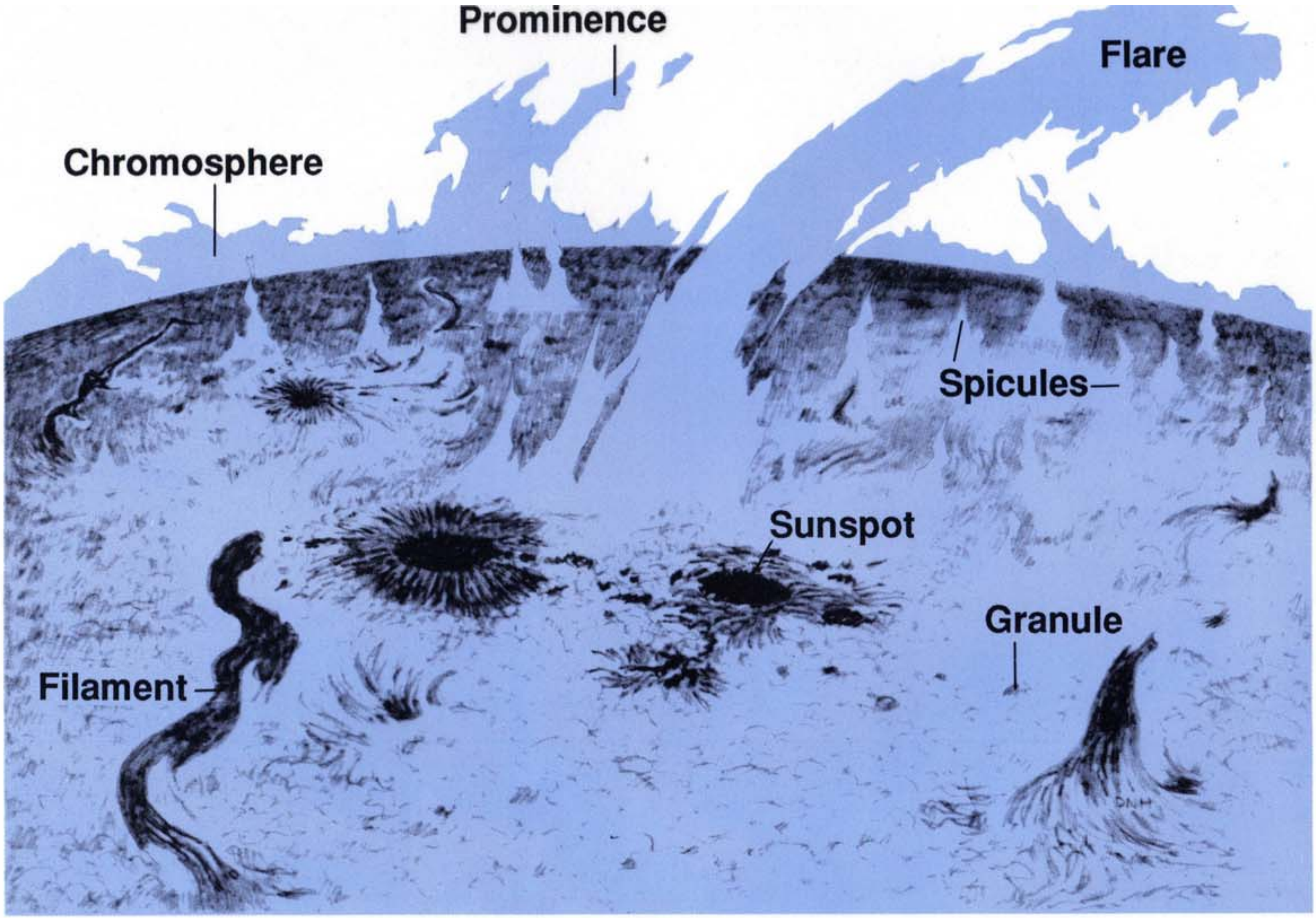
MSW effect → neutrinos oscillate among flavors due to electron interactions in Sun

$\nu_e \rightarrow \nu_\mu$ or ν_τ
electron muon tau

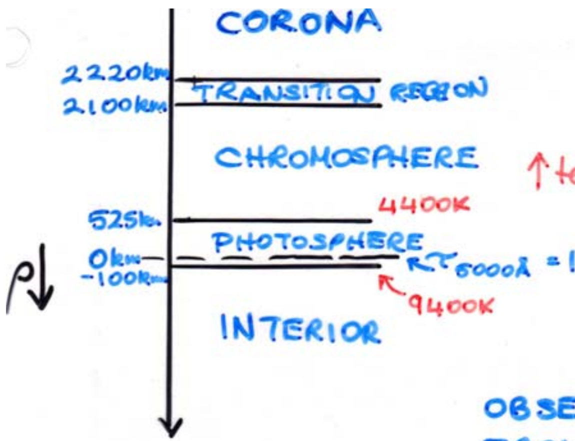
1998 Superkamiokande group detected ν_μ's consistent with MSW theory

All detectors sensitive only to ν_e

Suggests non-zero neutrino masses with $m(\nu_e) < m(\nu_\mu) < m(\nu_\tau)$


Prominence
Flare
Chromosphere
Spicules
Sunspot
Granule
Filament

Schematic of Sun's structure



-100km BELOW $\tau_{5000Å} = 1$

≡ BASE OF PHOTOSPHERE

OBSERVED OPTICAL PHOTONS FROM PHOTOSPHERE

$T_{eff} = 5777K$ AVERAGE OPTICAL DEPTH OF ORIGIN ≡ SURFACE AT $\tau = 2/3$

OPTICAL DEPTH → EXPONENTIAL FALL-OFF OF INTENSITY

IF ONLY 1% PHOTONS FROM SOME LEVEL REACH US,

FALL OFF IS $e^{-4.5} = 0.01$ AND $\tau = 4.5$

IF 0.01% ARRIVE, $\tau = 6.9$ SINCE $0.001 = e^{-6.9}$

Recall Planck-like emission spectrum ≈ BLACK BODY

CONSISTENT WITH CONTINUOUS SOURCE OF OPACITY

– FROM H^- IONS

PHOTONS WITH $E \geq hc/\lambda > X$ AFFECT

$\therefore \lambda < hc/0.75 < 170\mu m$

i.e FROM INFRARED TO LOWER WAVELENGTHS

CHROMOSPHERE

SEEN DURING ECLIPSE

3.

ρ DECREASES (by 10^4)

T INCREASES 4400K → 25,000K

LINES CAN FORM – IN EMISSION

SPICULES ~ 10,000 KM, 15 MINS

N ~ 30,000 AT ANY TIME

TRANSITION REGION ~ 10^6K

HIGH TEMPERATURES REQUIRED TO EXCITE ⇒ 1216Å Lyα, 20,000K

1032Å OVI 300,000K

625Å MgX 1.4×10^6K

CORONA

VISIBLE DURING TOTAL SOLAR ECLIPSE

10^5 ptcls/cm^3 (FAINTER THAN PHOTOSPHERE)

⇒ e.g. Fe X etc.

DIFFUSE ⇒ SOLAR WIND IONS & ELECTRONS

AT EARTH, $\rho \sim 10$ ptcles/cm^3 AT 500 km/sec

MASS LOSS RATE ~ $10^{-13} M_\odot$/year

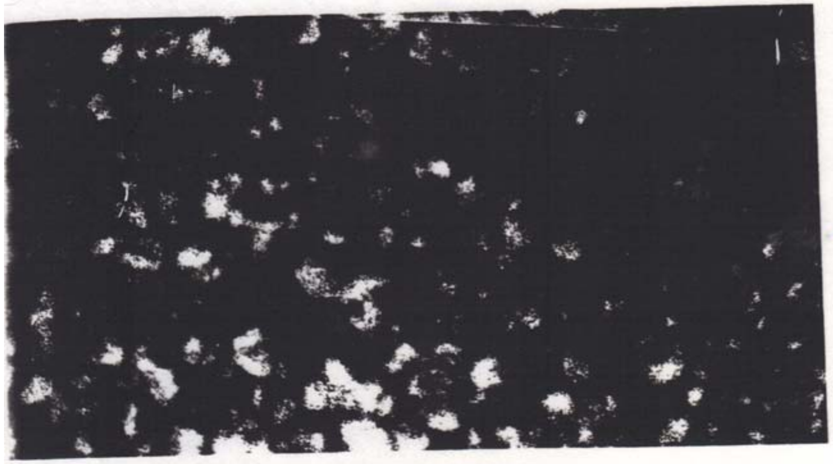
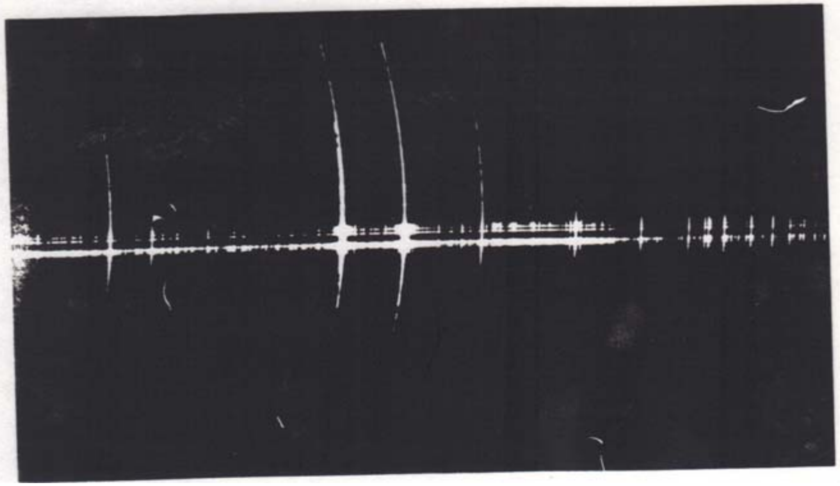


Fig. 13.3. The granulation of the solar surface. The granules are produced by streaming gas. Their typical diameter is 1000 km. (Photograph Mt. Wilson Observatory)

GRANULATION
- EVIDENCE FOR CONVECTION

During eclipses, the chromospheric spectrum, called the flash spectrum, can be observed. It is an emission line spectrum with more than 3000 identified lines. Brightest among these are the lines of hydrogen, helium and certain metals.

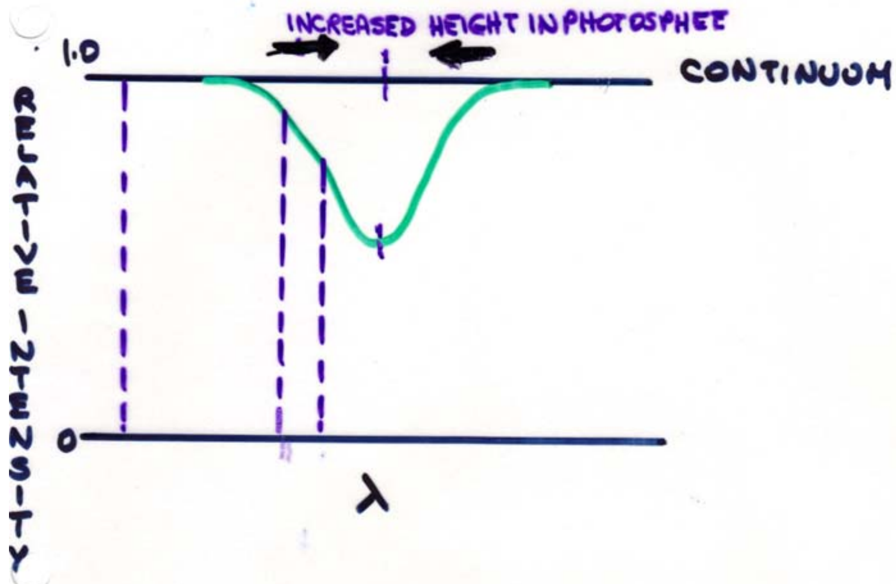
One of the strongest chromospheric emission lines is the hydrogen Balmer α line (Fig. 13.5) at a wavelength of 656.3 nm. Since the Hα line in the normal solar spectrum [is a] very dark absorption line, a photograph taken at this wavelength will show the [solar] chromosphere. For this purpose, one uses narrow-band filters letting through on[ly] the light in the Hα line. The resulting pictures show the solar surface as a mottled, wavy disc. The bright regions are usually the size of a supergranule, and are bounded by *spicules* (Fig. 13.6). These are flamelike structures, rising up to 10 000 km above the



CHROMOSPHERE
~ 500 km thick
(outside photosphere)
4500 K < T < 6000 K
Weak radiation visible only during eclipse

Fig. 13.4. Flash spectrum of the solar chromosphere, showing bright emission lines

SPECTRAL LINES VINDICATE SOLAR MODEL



- FRAUNHOFFER ABSORPTION SPECTRUM PRODUCED IN PHOTOSPHERE

ABSORPTION PRODUCED IN REGIONS COOLER THAN CONTINUUM-PRODUCING LEVELS (KIRCHHOFF)

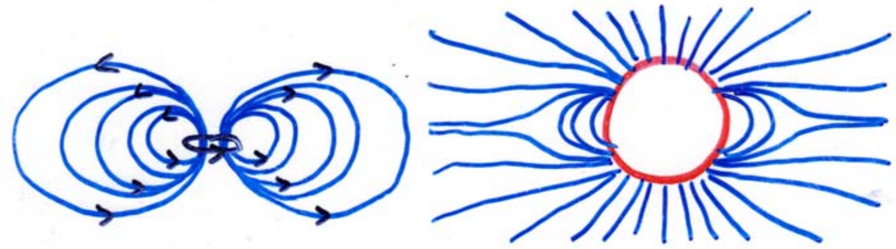
IN PRACTICE: IN SAME LEVELS
MIXTURE OF LINE FORMATION AND H^- CONTINUUM

BUT CENTER OF LINE FROM HIGHER LEVELS OF PHOTOSPHERE (WHERE OPACITY HIGHEST) WHICH ARE COOLER

⇒ IN WINGS OF LINE, EMISSION FROM DEEPER (EVENTUALLY FROM BASE OF PHOTOSPHERE)

– SPECTRAL LINES PROBE STELLAR STRUCTURE

OBSERVATIONS OF COMETS SUGGEST
SOLAR WIND, SOLAR MAGNETIC FIELD



DIPOLE FIELD FOR CURRENT LOOP

SUN'S MAGNETIC FIELD

- CHARGED PARTICLES – IONS/ELECTRONS SPIRAL AROUND FIELD LINES
- OPEN LINES ≡ WIND

COMETS → CURVED TAIL
+ STRAIGHT TAIL

STRAIGHT ≡ CHARGED PARTICLES
[INTERACTING WITH IONS OF SOLAR WIND]

CURVED ≡ DUST AT DIFFERENT ORBITAL SPEEDS [KEPLER'S LAWS]

[DUST PUSHED BACK BY RADIATION PRESSURE]