AY 20 Fall 2010

Stellar Interiors: Energy Sources

Reading: Carroll & Ostlie, Chapter 10 §10.3, §10.6

Last class: mean molecular weight of gas $\mu = \langle m \rangle / m_H$

$$\frac{1}{\mu_n} = \sum_j \frac{X_j}{A_j}$$ for completely neutral gas

$$\frac{1}{\mu_i} = \sum_j \frac{(1+z_j)X_j}{A_j}$$ for completely ionized gas

Where X_j = mass fraction for atoms of type j; $A_j = m_j/m_H$

Usual usage: X = total mass of H/total mass of gas

Y = total mass of He/total mass of gas

Z = total mass of metals/total mass of gas

Typically abundances are solar: X= 0.70 Y = 0.28. Z= 0.02

$$1/\mu_n = X + \tfrac{1}{4}Y + \langle 1/A \rangle_n Z$$

$$1/\mu_i = 2X + \tfrac{3}{4}Y + \langle (1+z)/A \rangle_i Z$$ (zj =# free electrons releasedby atom j)

$$1/\mu_i = 2X + \tfrac{3}{4}Y + \tfrac{1}{2}Z$$

Last two equations of stellar structure

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$ for radiative transport (1)

$$\frac{dT}{dr} = -\left(1-\frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2}$$ adiabatic convective transport (2)

convection condition for bubble: $d\rho/dr|_b < d\rho/dr|_{surroundings}$

$\rightarrow dT/dr|_{actual} < (1 - 1/\gamma)\ T/P \times dP/dr$

But (2) may be written $dT/dr|_{adiabatic} = (1 - 1/\gamma)\ T/P \times dP/dr$

$\therefore\ dT/dr|_{ad} > dT/dr|_{act}$

$\therefore$ since $dT/dr < 0$, condition for convection:

$|dT/dr|_{act} > |dT/dr|_{ad}$

i.e. actual temperature gradient is *superadiabatic*, assuming constant μ

Alternative condition: $d\ \ln P/d\ \ln T < \gamma/(\gamma-1) = 2.5$ for monatomic gas

For $d\ \ln P/d\ \ln T < 2.5$, convective transport – equation (2) above

For $d\ \ln P/d\ \ln T > 2.5$ radiative transport – equation (1) above

Equations of stellar structure

- equation of hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{GM_r\rho}{r^2}$$

- mass conservation equation

$$\frac{dM_r}{dr} = 4\pi r^2 \rho$$

- energy conservation equation

$$\frac{dL}{dr} = 4\pi r^2 \rho\varepsilon$$

- radiation transport

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2}$$

- adiabatic convection

$$\frac{dT}{dr} = -\left(1-\frac{1}{\gamma}\right)\frac{\mu m_H}{k}\frac{GM_r}{r^2}$$

LUMINOSITY GRADIENT EQUATION ⇒

$$\frac{dL_r}{dr} = 4\pi r^2 \rho \mathcal{E}$$

$\mathcal{E}$ = total energy released / unit mass / sec by all nuclear reactions + gravity

NUCLEAR REACTIONS – CONVERSION OF ONE ELEMENT TO ANOTHER

- RELEASES MeV ENERGIES
- CONVERSION VIA A CHAIN OF REACTIONS

CONSERVATION LAWS APPLY: CONSERVE

ELECTRIC CHARGE

NUMBER OF NUCLEONS

NUMBER OF LEPTONS

LEPTONS e^- e^+ ν - $\bar{\nu}$, γ

electron positron neutrino & anti-neutrino ← photon

NOMENCLATURE: $^{A}_{Z}X$

X = element; H, He, Ca ....

Z = number of protons (total +ve charge)

A = baryonic (or mass) number # of protons + neutrons

e.g. $^{1}_{1}P \equiv {}^{1}_{1}H$ hydrogen; $^{2}_{1}H$ = deuterium

neutrinos neutral + almost massless $^{0}_{0}\nu$

electron $^{0}_{-1}e$; positron $^{0}_{1}\bar{e}$ or $^{0}_{1}e^+$

5.

Showed earlier that nuclear energy sufficient to power Sun

Binding energy = energy released in fusion

e.g mass of 4H atoms > mass of He atoms

$$\Delta m = 0.028697\,u$$

$$E = \Delta mc^2 = 26.73\,\text{MeV}$$ - binding energy

Quantum tunneling enables fusion at temperatures typical of 'normal' stars but not all particles will have sufficient energy to tunnel even at these temperatures

∴ consider number density of ptcles. w. appropriate energies*

+ probability of tunneling through Coulomb barrier of target nucleus

↓

total nuclear reaction rates

* ie. in a specific range

? NUMBER DENSITY OF NUCLEI WITHIN SPECIFIED ENERGY INTERVAL? 6.

MAXWELL-BOLTZMANN DISTRIBUTION FUNCTION
(# OF PTCLES /UNIT VOL WITH SPEEDS BETWEEN v AND $v+dv$) AS FUNCTION OF T

$$n_v d_v = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}\, 4\pi v^2 dv$$

$E = K = \mu_m v^2/2$ (NEGLECT P.E. - ptcles INITIALLY FAR APART)

$$\Rightarrow n_E dE = \frac{2n}{\pi^{1/2}} \frac{1}{(kT)^{3/2}} E^{1/2} e^{-E/kT} dE$$

$n_E dE$ = NUMBER OF PARTICLES/UNIT VOL WITH ENERGIES BETWEEN E AND E+dE

WHAT IS PROBABILITY PARTICLES WILL INTERACT?

LET $\sigma(E) = \dfrac{\text{NUMBER OF REACTIONS/NUCLEUS/TIME}}{\text{NUMBER OF INCIDENT PTCLES/AREA/TIME}}$

CARROLL & OSTLIE PP. 303 - 306

⇓

TOTAL NUMBER OF REACTIONS/UNIT VOL/UNIT TIME = r_{ix}

$$r_{ix} = \int_0^\infty n_x n_i\, \sigma(E)\, v(E) \frac{n_E}{n} dE$$

n_i = number of incident particles, n_x = number of targets
$v(E) = \sqrt{2E/\mu_m}$, n_E number with energy E, $n = \int_0^\infty n_E dE$

AND $\sigma(E)$ CHANGES RAPIDLY WITH E

$\sigma \propto e^{-bE^{-1/2}}$

SIMPLE FORM:

$$r_{ix} = \left(\frac{2}{kT}\right)^{3/2} \frac{n_i n_x}{(\mu_m \pi)^{1/2}} \int_0^\infty S(E) e^{-bE^{-1/2}} e^{-E/kT} dE$$

$$\sigma(E) = S(E)/E\, e^{-bE^{-1/2}}, \quad b \equiv \frac{2^{3/2}\pi^2 \mu_m^{1/2} Z_1 Z_2 e^2}{h}$$

ENERGY RELEASE

FOR A 2-PARTICLE INTERACTION, REACTION RATES CAN BE EXPRESSED AS A POWER LAW CENTERED ON TEMPERATURE T

$$r_{ix} \simeq r_0 X_i X_x \rho^{\alpha'} T^{\beta}$$

WHERE r_0 = CONSTANT; $X_i X_x$ ARE MASS FRACTIONS OF TWO SPECIES

α', β DEPEND OF POWER LAW EXPANSIONS OF RATE EQUATIONS

$\alpha' = 2$ FOR 2-PARTICLE; $\beta = 1-40$

LET $\mathcal{E}_0$ = AMOUNT OF ENERGY PER REACTION

∴ ENERGY RELEASED/UNIT MASS/SEC = $\mathcal{E}_{ix} = \left(\frac{\mathcal{E}_0}{\rho}\right) r_{ix}$ *

$$\mathcal{E}_{ix} = \mathcal{E}_0' X_i X_x \rho^{\alpha} T^{\beta}, \quad \alpha = \alpha' - 1$$

watts/kg or ergs/sec/gm/...

FOR PP CHAIN: $\mathcal{E}_{pp} = 1.07 \times 10^{-5} \rho X^2 \psi_{pp} f_{pp} T_6^4$ cgs

$\psi_{pp} \sim 1$ to account for PPI, II, III; f_{pp} – "screening factor"; ionization → sea of -ve charges lowers Coulomb barrier

FOR CNO CYCLE: $\mathcal{E}_{CNO} = 8.24 \times 10^{-24} \rho X X_{CNO} T_6^{19.9}$

HIGHER TEMPERATURES AND MORE MASSIVE STARS } ENABLE CNO CYCLE

→ CLUES TO STRUCTURE OF STELLAR INTERIORS

* Note: total energy generation rate = $\Sigma \mathcal{E}_{ix}$ for all reactions

SINCE STARS MOSTLY HYDROGEN CONCENTRATE ON CONVERTING HYDROGEN TO HELIUM

- 1938 HANS BETHE (CORNELL) : CARL FRIERICH VON WEIZSÄCKER SUGGESTED CNO CYCLE
- 1950's TRIPLE-ALPHA REACTION (TO BURN He) & PROTON-PROTON CHAIN

EACH STEP: DIFFERENT COULOMB BARRIERS, σ, REACTION RATES

PPI (THERE ARE SEVERAL PATHS)

$$^1_1H + ^1_1H \rightarrow ^2_1H + ^0_1e^+ + ^0_0\nu$$

decay of proton to neutron involves "weak force"

THE SLOWEST STEP: INVOLVES $^1_1p \rightarrow ^1_0n + ^0_1e^+ + ^0_0\nu$

$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma$$

HELIUM-3

$$^3_2He + ^3_2He \rightarrow ^4_2He + 2\,^1_1H$$

IN SUN-LIKE STARS HAPPENS 69% OF TIME

$$\left(4\,^1_1H \rightarrow ^4_2He + 2\,^0_1e^+ + 2\,^0_0\nu + 2\gamma\right) \leftarrow \text{EVENTUALLY}$$

31% OF TIME PPII LIKELY

PPII BEGIN AS BEFORE

$$^3_2He + ^4_2He \rightarrow ^7_4Be + \gamma$$

$$^7_4Be + ^0_{-1}e \rightarrow ^7_3Li + ^0_0\nu$$

$$^7_3Li + ^1_1H \rightarrow 2\,^4_2He$$

PPIII

$$^7_4Be + ^1_1H \rightarrow ^8_5B + \gamma$$

$$^8_5B \rightarrow ^8_4Be + ^0_1e^+ + ^0_0\nu$$

$$^8_4Be \rightarrow 2\,^4_2He$$

IF PPII TAKES PLACE, PPIII 0.3% OF TIME (BRANCHING RATIO)

PROTON-PROTON CHAIN & BRANCHING RATIOS 19

$$^1_1H + ^1_1H \rightarrow ^2_1H + ^0_1e + ^0_0\nu \quad (1)$$

$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma \quad (2)$$

69% ↙ ↘ 31%

PPI $^3_2He + ^3_2He \rightarrow ^4_2He + 2\,^1_1H$ (3)

$^3_2He + ^4_2He \rightarrow ^7_4Be + \gamma$

97% ↙ ↓ 0.3%

PPIII

PPII $^7_4Be + e^- \rightarrow ^7_3Li + ^0_0\nu_e$

$^7_3Li + ^1_1H \rightarrow 2\,^4_2He$

$^7_4Be + ^1_1H \rightarrow ^8_5B + \gamma$

$^8_5B \rightarrow ^8_4Be + ^0_1e^+ + ^0_0\nu$

$^8_4Be \rightarrow 2\,^4_2He$

For each reaction (3), (1) & (2) take place twice
(1) has a very low (unmeasured) probability
– at solar temperatures proton-proton collision ~ 10^{10} years → deuteron

BUT IF FASTER SUN WOULD HAVE EXHAUSTED: ALL FUEL

(2) much faster → deuterium abundance low

For stars with $T < 20 \times 10^6$ K about mass of Sun
ρ, T appropriate for energy generation
by pp chain ~ 91% energy

For higher temps and masses $> 1.5\,M_\odot$
CARBON-NITROGEN-OXYGEN CYCLE (CNO CYCLE)
DOMINATES – more dependent on temperature

HELIUM ALSO PRODUCED FROM HYDROGEN USING CARBON, NITROGEN, OXYGEN

THE CNO CYCLE — BETHE 1938

$${}^{12}_{6}C + {}^{1}_{1}H \rightarrow {}^{13}_{7}N + \gamma \qquad (1)$$

$${}^{13}_{7}N \rightarrow {}^{13}_{6}C + {}^{0}_{1}e^{+} + {}^{0}_{0}\nu \qquad (2)$$

$${}^{13}_{6}C + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + \gamma \qquad (3)$$

$${}^{14}_{7}N + {}^{1}_{1}H \rightarrow {}^{15}_{8}O + \gamma \qquad (4)$$

$${}^{15}_{8}O \rightarrow {}^{15}_{7}N + {}^{0}_{1}e^{+} + {}^{0}_{0}\nu \qquad (5)$$

$${}^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{12}_{6}C + {}^{4}_{2}He \qquad (6)$$

0.04% TIME ↓

$${}^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{16}_{8}O + \gamma$$

$${}^{16}_{8}O + {}^{1}_{1}H \rightarrow {}^{17}_{9}F + \gamma$$

$${}^{17}_{9}F \rightarrow {}^{17}_{8}O + {}^{0}_{1}e^{+} + {}^{0}_{0}\nu$$

$${}^{17}_{8}O + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + {}^{4}_{2}He$$

HERE REACTION 4 IS SLOWEST AND SO DETERMINES RATE FOR CNO CYCLE. AT 20×10^{6} K, (4) HAS REACTION TIME 10^{6} YRS. FRACTION OF ENERGY RELEASED < PP CHAIN. MORE CARRIED AWAY BY NEUTRINOS quanta.

The second reaction, where a deuteron and a proton unite to form the helium isotope ^{3}He, is very fast compared to the preceding one. Thus the abundance of deuterons inside stars is very small.

The last step in the pp chain can take three different forms. The ppI chain shown above is the most probable one. In the Sun, 91% of the energy is produced by the ppI chain.

It is also possible for ^{3}He nuclei to unite into ^{4}He nuclei in two additional branches of the pp chain.

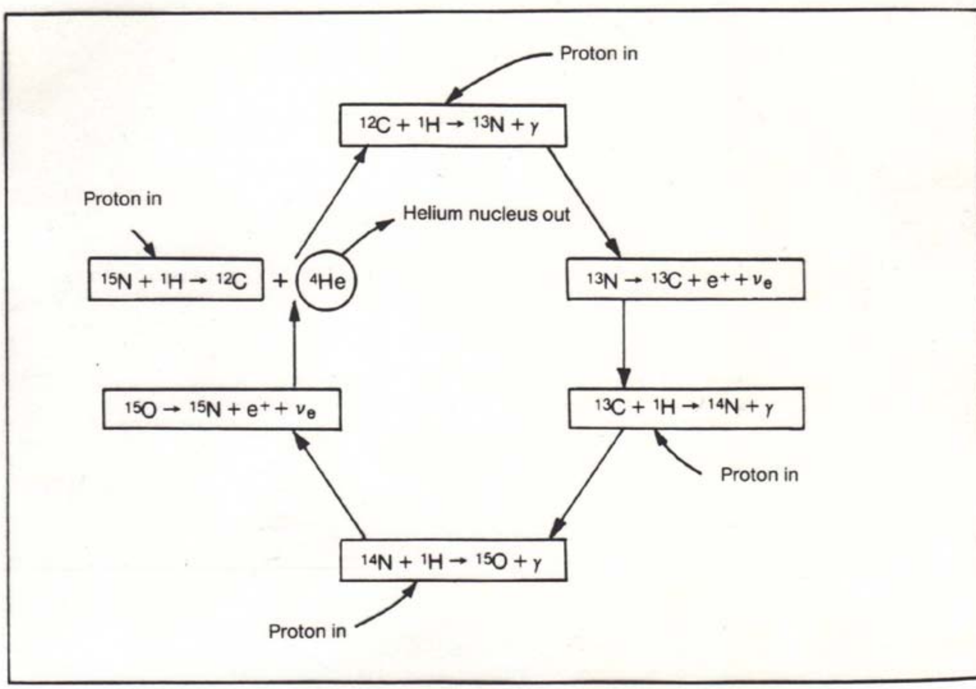
ppII:

(3) $^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$
(4) $^7\mathrm{Be} + \mathrm{e}^- \rightarrow {}^7\mathrm{Li} + \nu_e$
(5) $^7\mathrm{Li} + {}^1\mathrm{H} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$

ppIII:

(3) $^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$
(4) $^7\mathrm{Be} + {}^1\mathrm{H} \rightarrow {}^8\mathrm{B} + \gamma$
(5) $^8\mathrm{B} \rightarrow {}^8\mathrm{Be} + \mathrm{e}^+ + \nu_e$
(6) $^8\mathrm{Be} \rightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$.

The Carbon Cycle (Fig. 11.6). At temperatures below 20 million degrees, the pp chain is the main energy production mechanism. At higher temperatures, corresponding to



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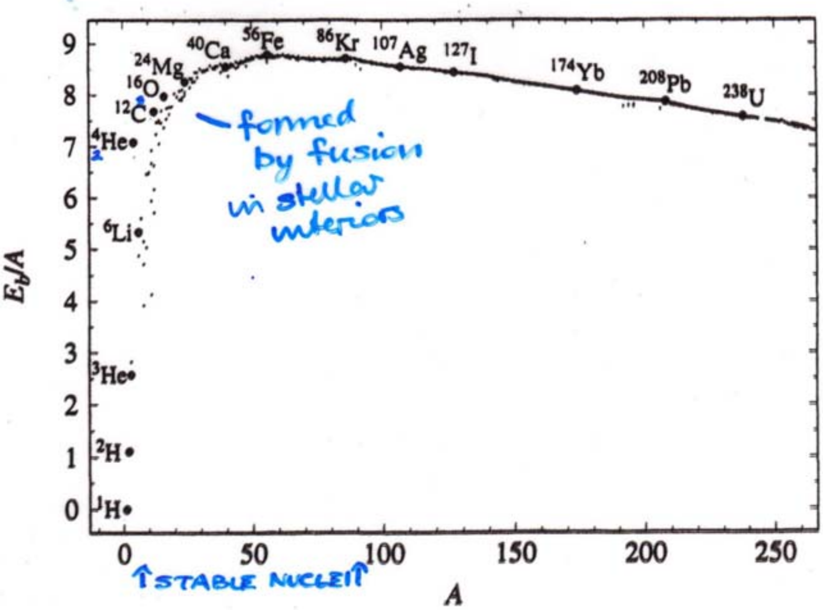
BINDING ENERGY/NUCLEON = E_b/A, A - mass number = m_i/m_H 13

$$E_b = \Delta m c^2 = (Z m_p + (A-Z) m_n - m_{nucleus}) c^2$$



ENERGY REGENERATION RATES:

PP CHAIN: $\epsilon_{pp} \simeq \epsilon'_{0,pp} \rho X^2 f_{pp} \psi_{pp} C_{pp} T_6^4$

$\epsilon'_{0,pp} = 1.08 \times 10^{-12}\ \mathrm{W\,m^3\,kg^{-2}}$ $\quad T_6 \equiv \frac{T}{10^6\,\mathrm{K}}$

low mass stars

$\epsilon_{pp} \propto T^4$

CNO CYCLE: $\epsilon_{CNO} = 8.67 \times 10^{20} \rho X X_{CNO} C_{CNO} T_6^{-2/3} e^{-152.28 T_6^{-1/3}}\ \mathrm{W\,kg^{-1}}$

$\epsilon_{CNO} \simeq \epsilon'_{0,CNO} \rho X X_{CNO} T_6^{19.9}$

hot, massive stars $> 1 M_\odot$

$\epsilon'_{CNO} = 8.24 \times 10^{-31}\ \mathrm{W\,m^3\,kg^{-2}}$

very temperature dependent

Conversion of H → He leads to increased μ

RECALL $\frac{1}{\mu_n} = X + \frac{1}{4}Y + \frac{1}{15.5}Z$

AS HYDROGEN → HELIUM, X DECREASES,
Y INCREASES, μ_n INCREASES

$PV = NRT \Rightarrow P_g = \frac{\rho k T}{\mu m_H}$

FOR CONSTANT ρ, T, INCREASED μ
≡ DECREASE P_g

REAL CHANGE, NOT EFFECT OF PRESSURE GRADIENT

∴ NO LONGER HYDROSTATIC EQUILIBRIUM

$dP/dr \neq -\frac{GM_r\rho}{r^2}$

∴ GRAVITY WINS, STAR COLLAPSE BEGINS

BUT COLLAPSE RAISES ρ, T

→ HELIUM "BURNING" CAN BEGIN

TEMPERATURES RISE TO POINT WHERE
HELIUM ATOMS CAN OVERCOME
COULOMB REPULSION

HELIUM CONVERTED TO CARBON IN
TRIPLE ALPHA PROCESS $T > 10^8 K$

$$\left.\begin{array}{l} {}^4_2He + {}^4_2He \rightleftarrows {}^8_4Be \\ {}^8_4Be + {}^4_2He \rightarrow {}^{12}_6C + \gamma \end{array}\right\} 3\,{}^4_2He \rightarrow {}^{12}_6C + \gamma$$

(IN 2×10^{-16} SECS)

BERYLLIUM WILL DECAY IF NOT HIT IMMEDIATELY BY ANOTHER α-PTCLE [HELIUM NUCLEUS]

$\therefore$ 3 BODY REACTION AND $\varepsilon_{3\alpha} \propto (\rho Y)^3$

$$\varepsilon_{3\alpha} \simeq \varepsilon'_{0,3\alpha}\, \rho^2 Y^3 f_{3\alpha} T_8^{41.0}$$

!!

REMARKABLE TEMPERATURE DEPENDENCE

IN THE BEGINNING ...... OR SOON AFTER BIG BANG

EARY UNIVERSE HYDROGEN, HELIUM

NOW: SIGNIFICANT ABUNDANCES METALS FROM NUCLEOSYNTHESIS IN STELLAR INTERIORS

ALPHA REACTIONS

${}^{12}_6C + {}^4_2He \rightarrow {}^{16}_8O + \gamma$

${}^{16}_8O + {}^4_2He \rightarrow {}^{20}_{10}Ne + \gamma$

${}^{20}_{10}Ne + {}^4_2He \rightarrow {}^{24}_{12}Mg + \gamma$

MORE MASSIVE STARS

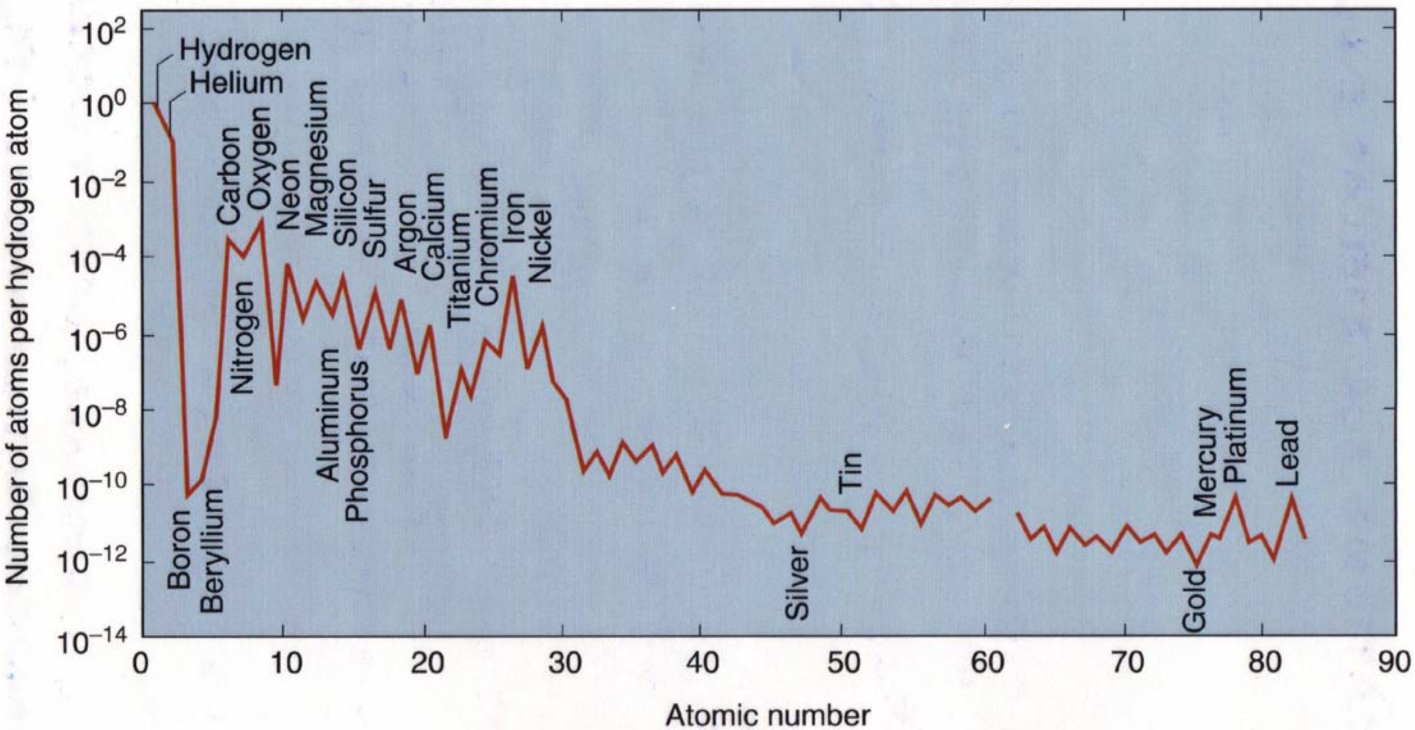
$5\text{-}8 \times 10^8 K$ CARBON BURNING → O

:

OXYGEN BURNING → various Si, Mg

Figure 13-1

Cosmic abundance of chemical elements



SOURCE: Data from A. Cameron.

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SUMMING UP:

FOR STARS OF MASS $< 1.5\, M_\odot$
TEMP $< 2 \times 10^7$ K
ENERGY GENERATION BY PP CHAIN

MORE MASSIVE, HOTTER STARS
ENERGY GENERATION BY CNO CYCLE

WITH INCREASING CENTRAL TEMPERATURE DUE TO HYDROGEN BURNING ($H \rightarrow He$)

$\downarrow$

BEGIN TO BURN HELIUM
- TRIPLE ALPHA PROCESS

MODELING STELLAR INTERIORS NOW POSSIBLE

DEFINE: BOUNDARY CONDITIONS,
USE: EQUATIONS OF STATE
STELLAR STRUCTURE EQUATIONS

SPECIFY: TYPE OF STAR
BY MASS, RADIUS, LUMINOSITY & COMPOSITION AT SURFACE

$\Rightarrow P, M_r, T, L_r$ AT dr BELOW SURFACE
etc etc etc $\rightarrow r = 0$

STELLAR MODELS RELY ON:

STRUCTURE EQUATIONS

$$\frac{dP}{dr} = -G\frac{M_r \rho}{r^2}, \quad \frac{dM_r}{dr} = 4\pi r^2 \rho, \quad \frac{dL_r}{dr} = 4\pi r^2 \rho \epsilon$$

$$\frac{dT}{dr} = -\frac{3}{4ac}\frac{\bar{\kappa}\rho}{T^3}\frac{L_r}{4\pi r^2} \text{ (radiation) } \quad \text{OR} = \left(1-\frac{1}{\gamma}\right)\frac{\mu m_H}{k} G\frac{M_r}{r^2}$$

(adiabatic convection)

SUPPLEMENTED BY "CONSTITUTIVE RELATIONS:"

$\equiv$ RELATIONS FOR P, $\bar{\kappa}$, ϵ, IN TERMS OF ρ, T, COMPOSITION

FOR PRESSURE: $P \approx \frac{\rho k T}{\mu m_H} + \frac{1}{3}aT^4$ VARIATION OF μ WITH COMPOSITION & IONIZATION MUST BE INCLUDED

(except in deep interior where ρ, T v. high)

OPACITY – NO ONE EQUATION; CALCULATED FOR VARIOUS COMPOSITIONS, ρ, T → TABLES → EXPRESSIONS

(not κ_{bb}) FOR κ_{bf}, κ_{ff} etc by extrapolation (or fitting fn.)

ENERGY GENERATION RATE: ϵ_{pp}, ϵ_{CNO} OF APPROPRIATE FORM

APPROPRIATE REACTIONS – DEPENDENT ON TEMP, DENSITY

– STRUCTURE EQUATIONS MUST BE INTEGRATED NUMERICALLY, NO ANALYTICAL SOLUTIONS (EXCEPTION POLYTROPES $P = K\rho^\gamma$)

– USE CONSTRUCT OF SPHERICALLY SYMMETRIC SHELLS AND DIFFERENCE EQUATIONS $\Delta P / \Delta r$

– USUALLY INTEGRATE SURFACE → IN & CENTER → OUT AND CONSTRAINED TO VARY SMOOTHLY AT "FITTING POINT"

BOUNDARY CONDITIONS → LIMITS OF INTEGRATION

PHYSICALLY CONSTRAIN EQUATIONS

∴ A STAR'S CENTER $M_r \to 0$ as $r \to 0$

$L_r \to$

AT SURFACE $T \to 0$ as $r \to R_*$

$P \to 0$

$\rho \to 0$

BUT STRUCTURE EQUATIONS P.18 AND CONSTITUTIVE REL^{N}'S ARE ALL INTER-RELATING:

$P, \bar{\kappa}, \epsilon$ DEPEND ON ρ, T, COMPOSITION AT GIVEN LOCATION

∴ IF $\mathcal{M}_r$ AT $r = R_*$ $(= \mathcal{M}_*)$ IS DEFINED/SPECIFIED AS WELL AS COMPOSITION, R_*, L_*

BOUNDARY CONDITIONS → P, M_r, T_r, L_r AT distance dr from surface

– CONTINUING NUMERICAL INTEGRATION TO CENTER → BOUNDARY CONDITIONS AT $r=0$

VALUES OF ALL GRADIENTS ALL INVOLVE COMPOSITION

∴ SURFACE RADIUS – LUMINOSITY COMBINATION DEFINED BY CHOICE OF MASS, COMPOSITION

⇒ VOGT-RUSSELL 'THEOREM'

MASS & COMPOSITION STRUCTURE THROUGHOUT A STAR UNIQUELY DETERMINE RADIUS, LUMINOSITY AND INTERNAL STRUCTURE AS WELL AS SUBSEQUENT EVOLUTION (NEGLECTING ROTATION & MAGNETIC FIELDS)

REALLY: STAR'S MASS AND COMPOSITION DICTATE EVOLUTION DUE TO CHANGES IN COMPOSITION FROM NUCLEAR BURNING

DUE TO CHANGES IN COMPOSITION & MASS FROM NUCLEAR BURNING, STAR EVOLVES

AS NUCLEAR BURNING PROCEEDS, STAR'S COMPOSITION CHANGES

[RECALL $\frac{1}{\mu_n}$ CHANGES : X DECREASES, Y INCREASES]

∴ LUMINOSITY AND RADIUS CHANGE

⇒ EVOLUTIONARY TRACKS ON HERTZSPRUNG-RUSSELL DIAGRAM

MOST STARS: $X = 0.70$ $Y = 0.28$ $Z = 0.02$

EASIEST ≡ FIRST REACTIONS $^{1}_{1}H \rightarrow {}^{4}_{2}He$

PP CHAIN, CNO CYCLE QUITE SLOW

∴ COMPOSITION CHANGES SLOW

∴ CHANGES IN T_{eff}, L_{R_*} ARE SLOW

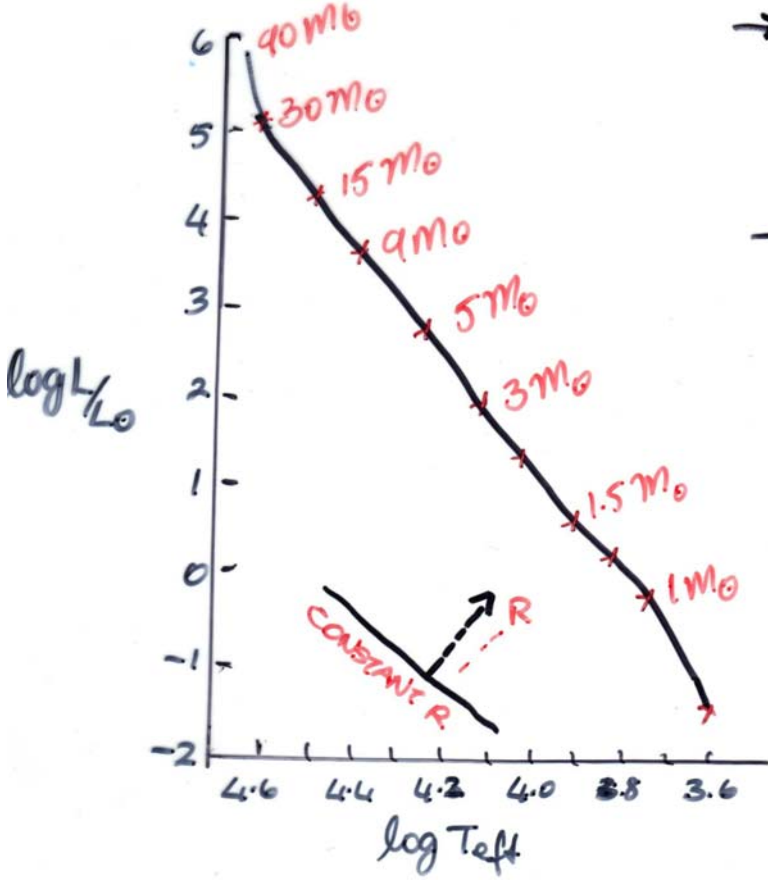
WHILE H-BURNING TAKING PLACE, LITTLE CHANGE IN OBSERVED CHARACTERISTICS OF STAR

WE SHOWED : $dP/dr = -\frac{GM_r\rho}{r^2} \Rightarrow P_c \sim G\frac{M_\odot\rho_\odot}{R_\odot}$

AND FOR GAS COMP$^{=}$ ONLY $T_c = \frac{P_c \mu m_H}{\rho k}$

∴ P_c, T_c, LUMINOSITY INCREASE WITH MASS

Plots of L versus T_{eff} for H-burning masses



→ mass* luminosity relation theoretically

→ H-burning on main-sequence

Masses determined from models

MORE MASSIVE ~ HOTTER ~ HIGHER L
REQUIRE GREATER ϵ

STARS WITH $M > 90\, M_\odot$ UNSTABLE

STARS WITH $M < 0.08\, M_\odot$ TOO COOL FOR NUCLEAR REACTIONS

EDDINGTON LIMIT:

EXTREMELY HIGH LUMINOSITIES
→ INSTABILITY OF MASSIVE STARS

RECALL $P = \frac{\rho k T}{\mu m_H} + \frac{1}{3} a T^4$

If T very high, ρ low $P_{rad} > P_{gas}$ outer layers massive ✱'s

∴ pressure grad is $\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c} F_{rad} = -\frac{\bar{\kappa}\rho}{c} \frac{L}{4\pi r^2}$

for hydrostatic equilibrium $\frac{dP}{dr} = -\frac{G M_r \rho}{r^2}$ $(-\rho g)$

∴ to maintain hydrostatic equilibrium

$$-\frac{\bar{\kappa}\rho}{c} \frac{L}{4\pi r^2} = -\frac{G M_r \rho}{r^2}$$

$$\therefore \quad L = \frac{4\pi c G M}{\bar{\kappa}} = L_{Eddington}$$

L_{ed} = maximum luminosity a star can have and still be in hydrostatic equilibrium

e.g. for massive star at 50,000K - upper end of main sequence
most H ionized $\bar{\kappa} \to \bar{\kappa}_{es} = 0.2(1+X)\ m^2 kg^{-1}$

$$\therefore L_{ed} = \frac{4\pi \times 6.7\times10^{-11} \times 3\times10^{8} \times 2\times10^{30}}{0.2\times1.7} \approx 1.5\times10^{31} \frac{M}{M_\odot}$$

$\frac{L_{Ed}}{L_\odot} = 3.8\times10^{4} \frac{M}{M_\odot}$ ∴ For $M = 90 M_\odot$

$L_{Ed} = 3.5\times10^{6} L_\odot$

expected m.s $L \approx 10^{6} L_\odot$

→ Envelopes of even m-s massive stars ~ unstable