

AY 20

Fall 2010

Stellar Atmospheres:
Spectral Line Profiles
Stellar Structure: Intro

Reading: Carroll & Ostlie, Chapters 9.5; 10.1; 10.2

Recap: Opacity

- Stars are not blackbodies: $\therefore T_e$ from $F_{\text{surface}} = \sigma T_{\text{eff}}^4$ not true *photospheric* temperature
- Stars are not in thermal equilibrium: use LTE concept
 - Applies if distance over which temperature changes significantly \gg mean free path of particles/photons (mean free path = $1/n\sigma$)
- Introduce opacity, κ_λ , $dI_\lambda = -\kappa_\lambda I_\lambda \rho ds \rightarrow I_\lambda = I_{\lambda,0} e^{-\int \kappa_\lambda \rho ds}$
 - pure absorption: intensity falls off exponentially (by e^{-1} at characteristic distance $\ell = 1/\kappa_\lambda \rho$)
 - scattering: ℓ = photon mean free path = $1/n\sigma_\lambda = 1/\kappa_\lambda \rho$
- Define optical depth, τ_λ , $d\tau_\lambda = -\kappa_\lambda \rho ds$; $I_\lambda = I_{\lambda,0} e^{-\tau_\lambda}$
 - s in direction of photon motion -outward; τ_λ inward from surface
 - since $\tau_\lambda = \kappa_\lambda \rho \int ds = \int ds/\ell$, optical depth = number of mean free paths from original position to surface

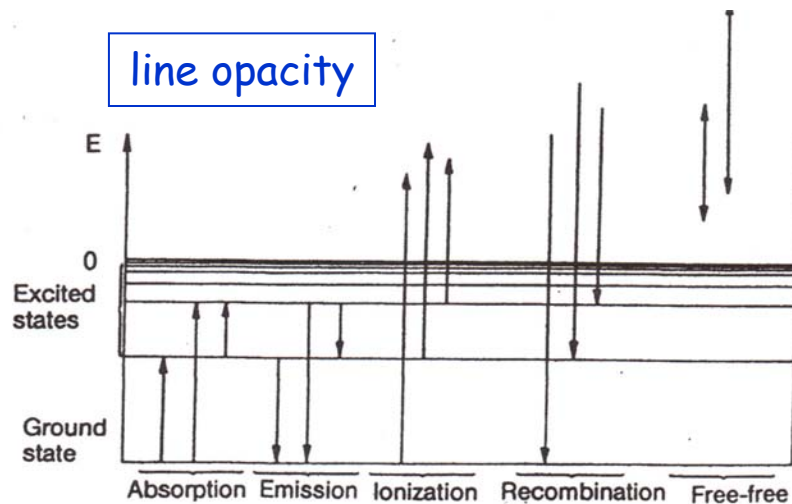
optically thick if $\tau_\lambda \gg 1$ optically thin if $\tau_\lambda \ll 1$

Sources of Opacity: slowly varying affects continuum; rapid variations → dark spectral lines

1. bound-bound transitions: photons "lost" to beam at discrete λ s
2. free-free transitions: absorption & bremsstrahlung - no preferred λ
3. bound-free transitions: photoionization* - any photon w. $\lambda < hc/\chi$
4. electron scattering:

* photoionization of H^- ions important continuum opacity source in stars cooler than F0
B and A stars: continuum opacity from photoioniz. of H atoms or free-free absorption
O stars: electron scattering and photoionization of He

Fig. 5.2. Different kinds of transitions between energy levels. Absorption and emission occur between two bound states, whereas ionization and recombination occur between a bound and a free state. Interaction of an atom with a free electron can result in a free-free transition



continuum opacity

5.2
"the H atom"

Recap: Radiative Transfer

- For stars, T and P_{rad} decrease outwards; $P_{\text{rad}} = 4\sigma T^4/3c$
 \rightarrow net flow of photons outwards
- introduce **emission coefficient j_λ** , analogous to κ_λ
 $dI_\lambda = -\kappa_\lambda \rho I_\lambda ds + j_\lambda \rho ds$ and $j_\lambda / \kappa_\lambda = S_\lambda$, **source function**
 $1/\kappa_\lambda \rho \times dI_\lambda/ds = I_\lambda - S_\lambda$ **transfer equation***
 in LTE, for optically thick gas, $S_\lambda = B_\lambda$
- a plane parallel, *gray*, atmosphere $\rightarrow \cos\theta dI_\lambda/d\tau_{\lambda,v} = I - S$
 $\tau_{\lambda,v}$ is vertical optical depth, *independent* of direction of "ray"
- expressing in terms of P_{rad} , F_{rad} :
 $dP_{\text{rad}}/d\tau_v = F_{\text{rad}}/c$ or $dP_{\text{rad}}/dr = -\kappa \rho F_{\text{rad}}/c$; (Rosseland mean κ)
 (net flux driven outwards by decreasing radiation pressure)
- integrating $dP_{\text{rad}}/d\tau_v$ with $P_{\text{rad}} = \frac{1}{3} = 4\pi/3c \langle I \rangle$ (Eddington approx)
 $T^4 = 3/4 T_{\text{eff}}^4 (\tau_v + 2/3)$, $T = T_{\text{eff}}$ at $\tau_v = 2/3$
 \therefore **photosphere at $\tau_v = 2/3$ (width $\sim 1\% R_*$)**

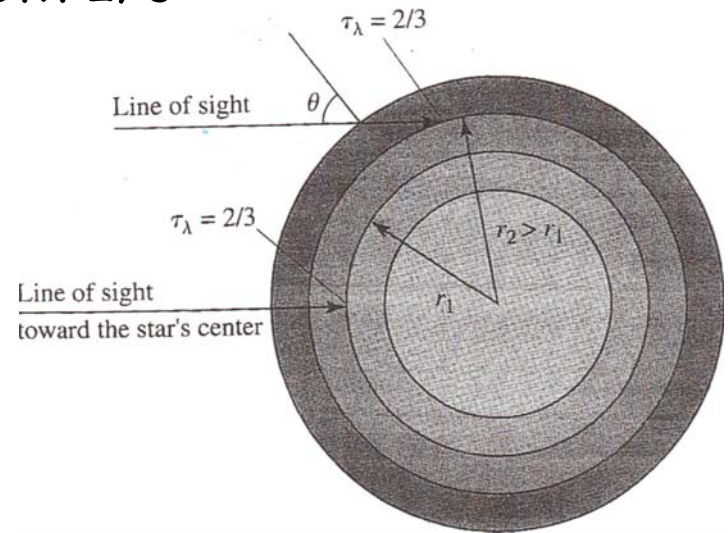
Note: for Sun, $T_{\text{eff}} = 5777\text{K}$. At $\tau_v = 0$, $T = 4852\text{K}$,

Effects of "seeing photosphere" at $\tau_v = 2/3$

≡ receiving radiation from optical depth 2/3

Line of sight at angle θ to emerging ray
 θ increases as approach limb

Observer sees vertically down into
Sun at center



With increasing θ , radial distance of a specific vertical depth in the atmosphere increases (= cooler)

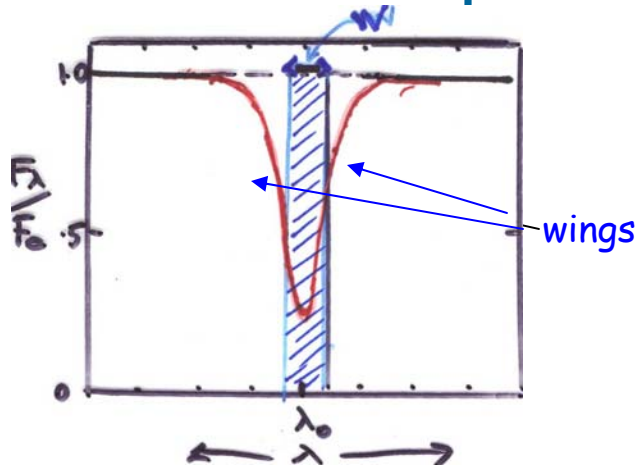
\therefore radiation from optical depth of 2/3 will come from regions of progressively lower temperature \rightarrow **limb darkening**

For stars, $\tau_\lambda = \kappa_\lambda \rho \int ds = 2/3$ at photosphere

As κ_λ increases, see in less deeply - to cooler levels

\rightarrow dark absorption lines: "profiles" - effect of layers of different temperatures

Line profiles: properties of stellar regions where absorption (b-b transitions) takes place



F_c = continuous spectrum flux level

F_λ = radiant flux from star

Line core at λ_0 = central wavelength

Line depth = $(F_c - F_\lambda) / F_c$

Strength of line = equivalent width
= width of box with same area as line

$W = \int [(F_c - F_\lambda) / F_c] d\lambda$, typically about 0.1 \AA

$(\Delta\lambda)_{1/2}$ = full width half maximum = width where depth = $\frac{1}{2}$

i.e. $[(F_c - F_\lambda) / (F_c - F_{\lambda,0})] = \frac{1}{2}$

F_λ always $> 0 \rightarrow$ optically thin line

maximum opacity κ_λ at λ_0 ; decreasing in wings

\therefore center of line forms in higher cooler part of atmosphere;

wings from increasingly deeper layers;

merge with continuum at optical depth $2/3$

Spectral lines have finite width

Various processes broaden lines
Produce characteristic line profiles

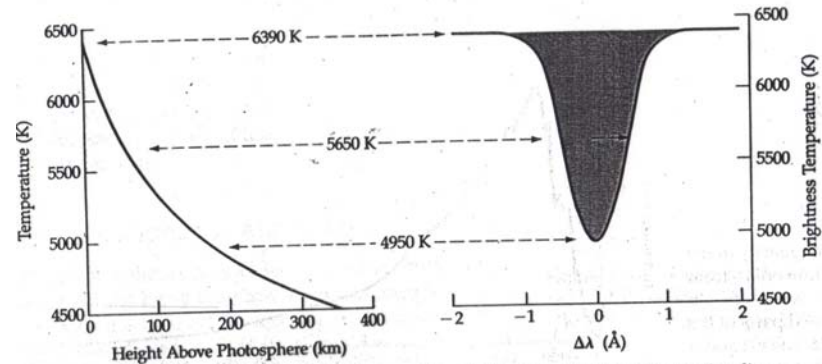


Figure 10-8 Photospheric depth and line profiles. Different parts of an absorption line form at different heights, the line center coming from the coolest, highest region. (R.W. Noyes)

Natural Broadening

Heisenberg's uncertainty principle; $\Delta x \Delta p \approx \hbar$ or $\Delta E \Delta t \approx \hbar \rightarrow \Delta E \approx \hbar / \Delta t$

Δt = lifetime in given state

\therefore Energy levels are "fuzzy" rather than precise and for transitions between levels, photon wavelength is 'fuzzy' ($E_{\text{photon}} \approx hc/\lambda$)

Uncertainty in wavelength $\Delta \lambda \approx \lambda^2 / 2\pi c (1/\Delta t_i + 1/\Delta t_f)$

Natural broadening $\sim 5 \times 10^{-4} \text{ \AA}$ for $H\alpha$

and $\text{FWHM} = \lambda^2 / \pi c (1/\Delta t_0) \sim 2 \times 10^{-4} \text{ \AA}$

Doppler Broadening

- for a gas in thermal equilibrium, velocities of atoms follow Maxwell-Boltzmann distribution, $v_{mp} = (2kT/m)^{1/2}$
- Doppler shift of wavelengths of absorbed/emitted radiation given by $\Delta\lambda/\lambda = \pm v_r/c$ (non relativistic Doppler shift)

$$\therefore \Delta\lambda \approx 2\lambda/c (2kT/m)^{1/2}$$

- Width due to Doppler broadening $\sim 0.4 \text{ \AA}$
and FWHM $= 2\lambda/c (2kT\ln 2/m)^{1/2}$

Much bigger effect than natural broadening
but line depth decreases much faster - exponentially
Applies to large-scale turbulent motions if these follow M-B
distribution

e.g. giants and supergiants display very broad lines

Pressure broadening (collisional broadening)

Collisions between atoms - collisional broadening

Statistical effects of close encounter with electric fields of many ions - pressure broadening

Like natural broadening $\Delta\lambda \approx \lambda^2/\pi c(\Delta t_0)$

Δt_0 = average time between collisions =

mean free path/velocity = ℓ/v

$$= 1/n\sigma(2kT/m)^{-1/2}$$

$$\therefore \Delta\lambda = \lambda^2/c \times n\sigma/\pi \times (2kT/m)^{-1/2}$$

$\Delta\lambda \propto n$ = number density of atoms

Hence MKK classes: Narrower lines in most luminous supergiants, giants due to diffuse atmospheres.

Pressure broadened lines in main-sequence stars due to more collisions

Voigt Profiles

Profile due to natural and pressure broadening:

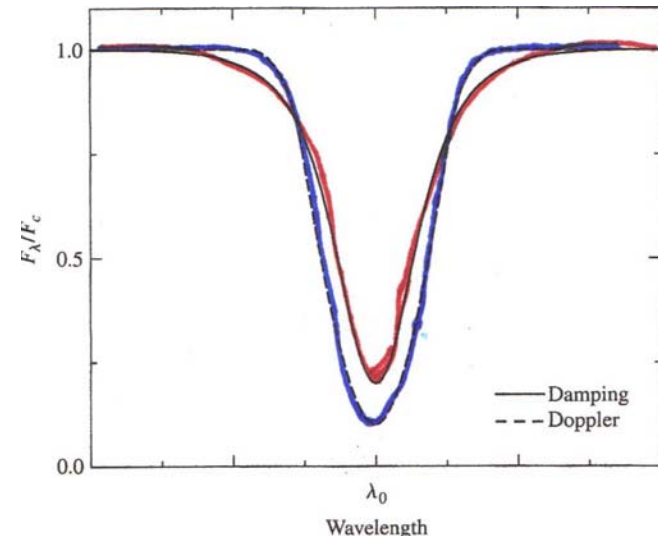
→ damping profile (cf damped harmonic motion of electric charge)

Voigt profile combines damping and Doppler profiles

- Doppler core and damping wings

Determine N_a , number of atoms/unit area with appropriate conditions for absorbing photon at λ of spectral line

Curves of growth and $W \propto N_a \rightarrow$ element abundances



Stellar Rotation and Surface Gravity also affect

176 17 Stellar rotation

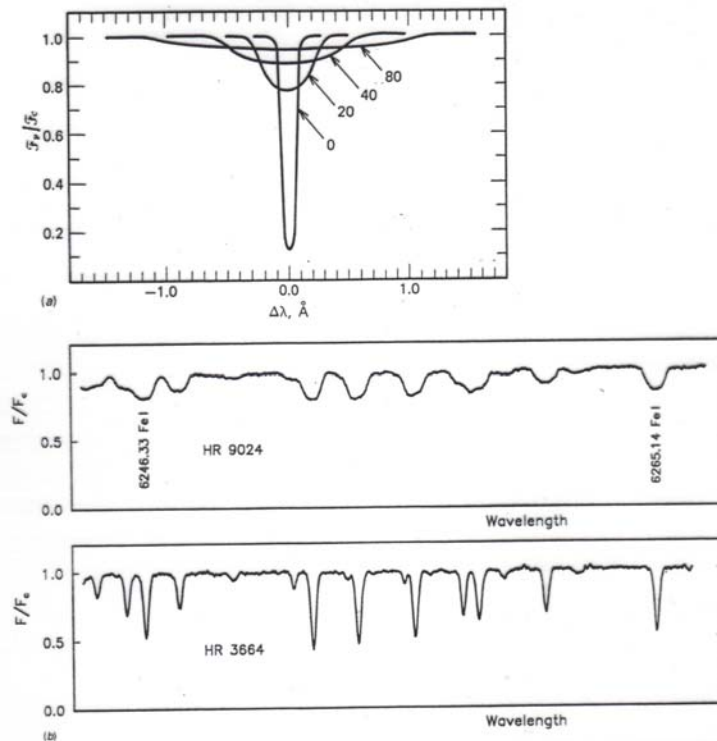


Fig. 17.7. (a) Computed profiles illustrate the broadening effect of rotation. The profiles are labeled with $v \sin i$, the wavelength is 4243 Å, and the line has an equivalent width of 100 mÅ. (b) These two early-G giants illustrate the Doppler broadening of the line profiles by rotation.

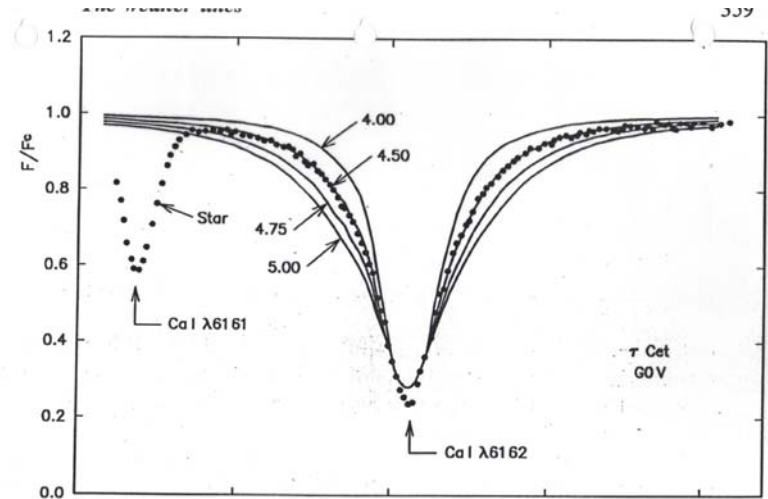


Fig. 16.5. Strong lines like this calcium I $\lambda 6162$ line have been used to measure surface gravity. These models indicated the surface gravity of τ Cet to be near $\log g = 4.5$. (Data from Smith and Drake (1987).)

Next year's
Ay classes!