AY 20 Fall 2010

Stellar Atmospheres: Radiative Transfer

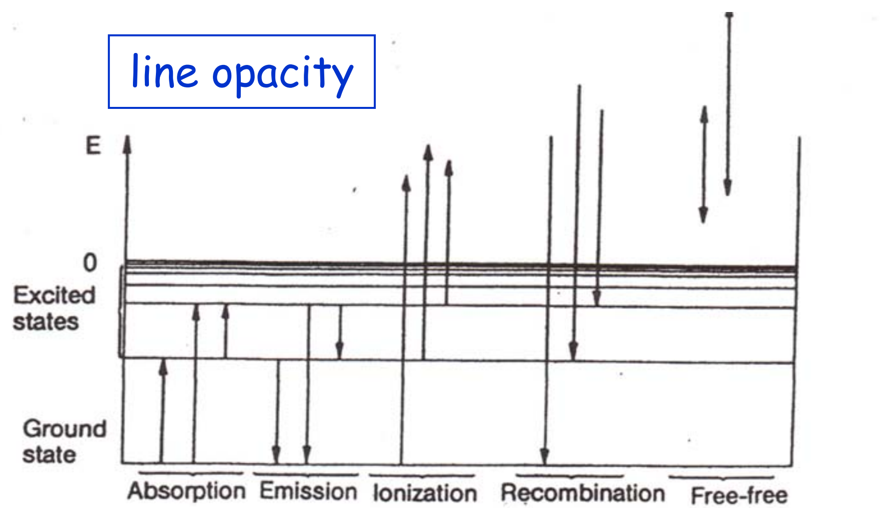
Reading: Carroll & Ostlie, Chapter 9.2, 9.3,9.4

Sources of Opacity: slowly varying affects continuum; rapid variations → dark spectral lines

1. bound-bound transitions: photons "lost" to beam at discrete λs
2. free-free transitions: absorption & bremsstrahlung – no preferred λ
3. bound-free transitions: photoionization* – any photon w. $\lambda < hc/\chi$
4. electron scattering: Thompson scattering at high T, ρ; also Compton or Rayleigh scattering

* photoionization of H^- ions important continuum opacity source in stars cooler than F0
B and A stars: continuum opacity from photoioniz. of H atoms or free-free absorption
O stars: electron scattering and photoionization of He

Fig. 5.2. Different kinds of transitions between energy levels. Absorption and emission occur between two bound states, whereas ionization and recombination occur between a bound and a free state. Interaction of an atom with an free electron can result in a free-free transition



continuum opacity

5.2
the H atom)

EFFECTS OF OPACITY

Recall for energy levels of H, cf §53

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

∴ for Balmer series $E_2 = -\frac{13.6}{4} = -3.4$ eV

∴ to ionize / eject electron, photon energy > 3.4 eV

Now $\lambda \leq hc/\chi_2$ to ionize H in first excited (n=2) state

$$\therefore \lambda < \frac{6.6\times10^{-34} \times 3\times10^{8}}{3.4 \times 1.6\times10^{-19}} \sim \frac{6}{1.6}\times10^{-7}$$

1.1

$\lambda \sim 3.7\times10^{-7}$ m ~ 3700 Å [3647 Å]

3647 nm

∴ any photon w. energy equivalent to $\lambda = 3647$ Å or less

(ie $\lambda <$ 3647 Å photons) – can ionize from n=2 level of H

κ_{bf} increases suddenly at 3647 Å

→ sudden drop in radiative flux due to increased opacity → **BALMER-JUMP**

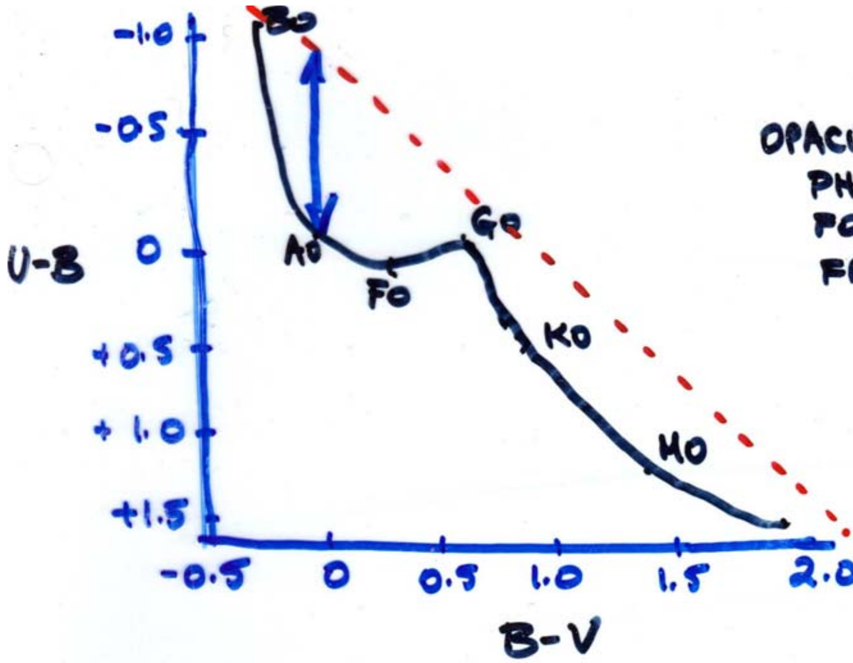
size of jump ~ FRACTION # of atoms in first excited state

∴ from Boltzmann equation

→ Temperature dependence

∴ <u>size of Balmer jump ∝ temperature</u>

(not for very hot or very cool)



OPACITY →
PHYSICAL REASONS
FOR DEVIATION
FROM B-B

U FILTER CENTERED AT 3650 Å

BALMER JUMP ~ 3647 Å

∴ FLUX IN BAND DECREASES
⇒ U (MAGNITUDES) INCREASES
⇒ U-B INCREASES

STRONGEST WHEN N_2/N_{tot} MAXIMUM (9600K)

9600K ≡ A0

= GREATEST DEVIATION
FROM B-B AT
SPECTRAL TYPE A0

SOURCES OF OPACITY DIFFERENT AT DIFFERENT TEMPERATURES

- MAJOR CONTRIBUTOR IN STARS COOLER THAN F0 IS H^- ION PHOTOIONIZATION

$\equiv$ SECOND ELECTRON BOUND TO ATOM (LOOSELY) OPPOSITE FIRST ELECTRON

BINDING ENERGY = 0.754 eV (13.6 eV FOR H ground state)

ANY photon with $\lambda < hc/\chi = \dfrac{6.6\times10^{-27}\times3\times10^{10}}{.75\times1.6\times10^{-12}}$

can photoionize ($\lesssim 1.6\,\mu m$)

$\rightarrow$ BOUND-FREE ABSORPTION CONTRIBUTES TO CONTINUUM OPACITY IN LATER-TYPE STARS

INCREASING TEMPERATURE $\rightarrow$ A, B STARS

& H^- INCREASINGLY IONIZED

CONTINUUM OPACITY FROM PHOTOIONIZATION OF H ATOMS κ_{bf}

and FREE-FREE ABSORPTION κ_{ff}

O STARS: PROGRESSIVELY MORE IONIZATION OF H SO ELECTRON SCATTERING κ_{es} + He ionization κ_{bf}

$$\therefore \kappa_\lambda = \kappa_{\lambda,bb} + \kappa_{\lambda,bf} + \kappa_{\lambda,ff} + \kappa_{es} + \kappa_{H^-}$$

very important

OPACITY ALSO DEPENDS ON DENSITY, COMPOSITION, TEMPERATURE

(– NOT JUST ON WAVELENGTH OF PHOTONS BEING ABSORBED)

ROSSELAND MEAN OPACITY, $\bar{\kappa}$, INDEPENDENT OF λ

– DEPENDS ONLY ON COMPOSITION, DENSITY TEMPERATURE

DEFINE $$\frac{1}{\bar{\kappa}} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu(T)}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu(T)}{\partial T} d\nu}$$

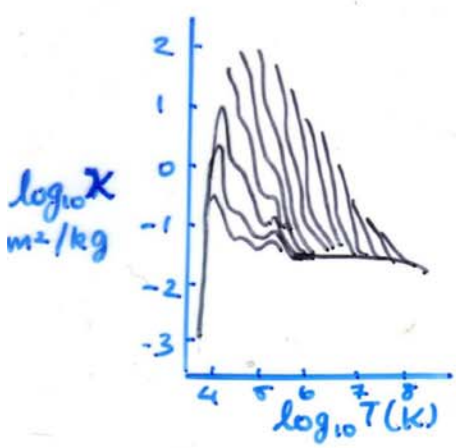
harmonic mean → lowest weight to high values of opacity

No analytic expression for $\bar{\kappa}_{bb}$ but simpler expressions for $\bar{\kappa}_{bf}$, $\bar{\kappa}_{ff}$, $\bar{\kappa}_{es}$, $\bar{\kappa}_{H^-}$ *

$$\bar{\kappa} = \overline{\kappa_{bb} + \kappa_{bf} + \kappa_{ff} + \kappa_{es} + \kappa_{H^-}}$$

* $\bar{\kappa}_{bf} = 4.34 \times 10^{21}\, g_{bf}/t\, Z(1+X)\, \rho/T^{3.5}$ m²/kg

$\bar{\kappa}_{ff} = 3.68 \times 10^{18}\, g_{ff}(1-Z)(1+X)\, \rho/T^{3.5}$ m²/kg



Rosseland Mean from computer calculation for composition 70% H, 28% He, 2% metals by mass

Curves are density (kg m⁻³)

Note $\bar{\kappa}_{es}$ independent of wavelength $= 0.02(1+X)$ m²/kg → all curves converge as T→

κ increases w increasing ρ for given T

steep rise in constant ρ plot with T

≡ increase in free electrons from ionization of H, He

fall-off as opacity dominated by κ_{bf}, κ_{ff}

10.3

OPACITY - MEASURE OF WHAT STOPS RADIATION FROM PROPAGATING THROUGH STELLAR ATMOSPHERE

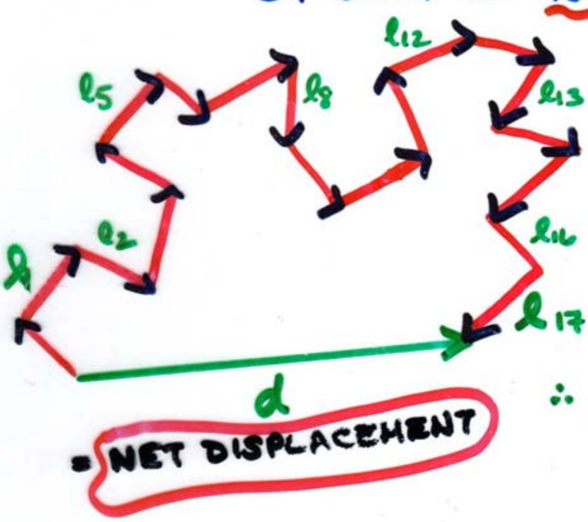
RADIATIVE TRANSFER - HOW RADIATION PROPAGATES

EQUILIBRIUM ≡ NO CHANGE IN TOTAL ENERGY IN A LEVEL

→ ABSORPTION & EMISSION BALANCE

→ PHOTONS DO NOT STREAM OUT - ABSORPTION & EMISSION REDIRECT, REDISTRIBUTE ENERGY

IN EFFECT, PHOTONS DIFFUSE UPWARDS IN ATMOSPHERE BY WAY OF RANDOM WALK



EACH STEP = l
= MEAN FREE PATH
$[l = 1/\kappa_\lambda \rho]$

$\underline{d} = \underline{l}_1 + \underline{l}_2 + \underline{l}_3 \dots \underline{l}_N$

$\therefore \underline{d}.\underline{d} = \underline{l}_1.\underline{l}_1 + \underline{l}_1.\underline{l}_2 + \dots \underline{l}_1.\underline{l}_N$
$+ \underline{l}_2.\underline{l}_1 \quad \dots \quad \underline{l}_2.\underline{l}_N$
$+ \dots \dots \quad l_N.l_N$

$\therefore \underline{d}.\underline{d} = \sum_{i=1}^{N}\sum_{j=1}^{N} \underline{l}_i.\underline{l}_j$ where N = number of steps

$\therefore d^2 = Nl^2 + l^2 [\cos\theta_{12} + \cos\theta_{13} + \dots \cos\theta_{1N} + \cos\theta_{21} + \dots + \cos\theta_{2N} + \dots \cos\theta_{N1} + \dots]$

FOR VERY LARGE N, $\Sigma \cos\theta$ TERMS $\to 0$

$\therefore d = l\sqrt{N}$

$$d = \ell\sqrt{N}$$

NUMBER OF STEPS

DISPLACEMENT MEAN FREE PATH

i.e. DISPLACEMENT $\propto \sqrt{\text{NUMBER OF STEPS}}$

→ TO MOVE $10 \times d$, NEED 100 MORE STEPS

ENERGY TRANSPORT IS <u>NOT</u> EFFICIENT!

RECALL: OPTICAL DEPTH, τ_λ

= NUMBER OF PHOTON MEAN FREE PATHS FROM POINT OF MEASUREMENT WITHIN STAR TO STELLAR SURFACE

$$\tau_\lambda = \int_0^s ds/\ell = d/\ell$$

$$\therefore d = \ell\tau_\lambda = \ell\sqrt{N}$$

$\therefore$ AVERAGE NUMBER OF STEPS TO REACH SURFACE

$$N = \tau_\lambda^2 \quad \text{assuming } \tau_\lambda \gg 1$$

& FOR $\tau \simeq 1$, PHOTON CAN ESCAPE

WE WILL SHOW THAT STELLAR PHOTOSPHERE, BY DEFINITION LAYER FROM WHICH VISIBLE LIGHT COMES, IS AT $\tau_\lambda \simeq 2/3$

WITHIN STAR, ANY PHOTON MOVES IN NEARLY RANDOM DIRECTION DUE TO MULTIPLE SCATTERINGS

BUT: TEMPERATURE DECREASES OUTWARD

$\therefore$ RADIATION PRESSURE DECREASES

RECALL $P_{rad} = \frac{4\pi}{3c}\int_0^\infty B_\lambda(T)\,d\lambda = \frac{4\sigma T^4}{3c}$

GRADIENT IN $P_{rad} \rightarrow$ NET MOTION OF PHOTONS TO SURFACE

$\rightarrow$ NET FLOW OF RADIATIVE FLUX

$\equiv$ slow upward diffusion of randomly moving photons

"BEAM" OR "RAY"

$\equiv$ DIRECTION OF MOTION SHARED AT PARTICULAR INSTANT BY PHOTONS BEING ABSORBED, SCATTERED INTO AND OUT OF "BEAM"

$\equiv$ "CONVENIENT FICTION"

NET FLOW OF RADIATION THROUGH ATMOSPHERE FROM EQUATION OF RADIATIVE TRANSFER

(NOTE: NOT SPECIFIC PATHS OF INDIVIDUAL PHOTONS)

SUPPOSE SOME EMISSION PROCESS INCREASES I_λ (INTENSITY) OF LIGHT AS IT MOVES THROUGH GAS:

RECALL ABSORPTION $dI_\lambda = -k_\lambda \rho I_\lambda ds$

SIMILARLY, FOR EMISSION $dI_\lambda = j_\lambda \rho ds$

j_λ = EMISSION COEFFICIENT

= ENERGY EMITTED BETWEEN λ & $\lambda + d\lambda$ INTO UNIT SOLID ANGLE/UNIT VOL/UNIT TIME

$$\therefore dI_\lambda = \underbrace{-k_\lambda \rho I_\lambda ds}_{\text{DECREASE}} + \underbrace{j_\lambda \rho ds}_{\text{INCREASE}}$$

$$\therefore -\frac{1}{k_\lambda \rho}\frac{dI_\lambda}{ds} = I_\lambda - j_\lambda/k_\lambda \quad (\text{or } j_\nu/k_\nu)$$

where $j_\lambda/k_\lambda \equiv S_\lambda$, the SOURCE FUNCTION

= ratio of emission coeff to absorption coeff

≡ how photons in beam are removed, replaced by local photons

$$\therefore \frac{1}{k_\lambda \rho}\frac{dI_\lambda}{ds} = I_\lambda - S_\lambda$$

TRANSFER EQUATION

S, I_λ same units - watts $m^{-2} sr^{-1}$

Since $-\frac{1}{\kappa_{\lambda}\rho}\frac{dI_\lambda}{ds} = I_\lambda - S_\lambda$

FOR NO VARIATION IN INTENSITY, $I_\lambda = S$

IF $I_\lambda > S$, I_λ DECREASES WITH DISTANCE

$I_\lambda < S$ I_λ INCREASES " "

$\Rightarrow$ I_λ TENDS TO S_λ $\equiv$ photons in beam tend to become like local source of photons

FOR BLACK-BODY RADIATION: Suppose "box" of optically thick gas at temperature T, no net flow of energy - all processes balance

$\therefore \frac{dI_\lambda}{ds} = 0$ so that $I_\lambda = S_\lambda = B_\lambda$ (since B-B)

$\therefore S_\lambda = B_\lambda$ FOR THERMODYNAMIC EQUILIBRIUM

FOR REAL STAR?

AT LEVEL WHERE $\tau_\lambda >> 1$, PHOTON NEEDS τ_λ^2 STEPS TO REACH SURFACE

i.e. PHOTON, mean free path, $\ell << H$, scale height

AND PHOTONS CONFINED TO LIMITED VOLUME AT TEMP CONSTANT

$\equiv$ LTE

i.e. $S_\lambda = B_\lambda$ in LTE with $\tau_\lambda >> 1$

i.e. the source of the radiation $j_\lambda / \kappa_\lambda$ results in B-B radiation

I_λ tends to S_λ

TO UNDERSTAND CONDITIONS IN STELLAR ATMOSPHERE NEED TO KNOW WHAT DEPTH SPECTRAL LINE FORMED

i.e NEED SOME MEASURE OF POSITION

REWRITE TRANSFER EQUATION:

$$-\frac{1}{K_\lambda \rho}\frac{dI_\lambda}{ds} = I_\lambda - S_\lambda \Rightarrow \frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

ASSUME ATMOSPHERIC LAYER IS THIN WITH NEGLIBLE CURVATURE

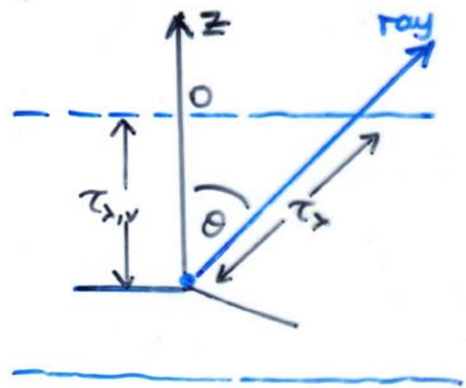
$\Rightarrow$ PLANE PARALLEL ATMOSPHERE

z - axis vertical, z = 0 at top

vertical optical depth

$$\tau_{\lambda,v}(z) \equiv \int_z^0 K_\lambda \rho \, dz$$

= starting optical depth for ray travelling to z=0 where $\tau_{\lambda,v} = 0$



$\tau_\lambda > \tau_{\lambda,v}(z)$ since path longer

and $\tau_\lambda = \frac{\tau_{\lambda,v}}{\cos\theta} = \tau_{\lambda,v}\sec\theta$

$\tau_{\lambda,v}$ independent of direction of light ray

and $\therefore$ $\cos\theta \frac{dI_\lambda}{d\tau_{\lambda,v}} = I_\lambda - S_\lambda$

If opacity $K_\lambda = \bar{K}$ - ROSSELAND MEAN (INDEPENDENT OF λ) $\Rightarrow \tau_v$

$$\therefore \cos\theta \frac{dI_\lambda}{d\tau_v} = \int_0^\infty I_\lambda d\lambda - \int_0^\infty S_\lambda d\lambda = I - S$$

TRANSFER EQUATION FOR GRAY ATMOSPHERE (i.e. opacity independent of λ)

FOR PLANE-PARALLEL GRAY* ATMOSPHERE

$$\cos\theta \frac{dI}{d\tau_v} = I - S$$

(* H^- PHOTOIONIZATION MAIN SOURCE OF OPACITY OVER WIDE RANGE OF λ's)

INTEGRATE OVER ALL SOLID ANGLES $\therefore \frac{d}{d\tau_v}\int I\cos\theta\, d\Omega = \int d\Omega\, I - S\int d\Omega$ (1)

RECALL $F_\lambda d\lambda = \int I_\lambda d\lambda \cos\theta\, d\Omega$, $\langle I\rangle \equiv \frac{1}{4\pi}\int I_\lambda d\Omega$ & $\int d\Omega = 4\pi$

$$\therefore \frac{dF_{rad}}{d\tau_v} = 4\pi(\langle I\rangle - S) \qquad (2)$$

MULTIPLY (1) BY $\cos\theta$

$$\therefore \frac{d}{d\tau_v}\int I\cos^2\theta\, d\Omega = \int I\cos\theta\, d\Omega - \int S\cos\theta\, d\Omega$$

RECALL $P_{rad,\lambda} d\lambda = \frac{1}{c}\int_{sphere} I_\lambda d\lambda \cos^2\theta\, d\Omega$, $F_{rad} = \int I\cos\theta\, d\Omega$

and $\int \cos\theta\, d\Omega = \int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta\, d\theta\, d\phi = 0$

$$\therefore \frac{dP_{rad}}{d\tau_v} = \frac{1}{c} F_{rad}$$

for spherical coords: with origin at star center $\frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c} F_{rad}$ (3)

⇒ NET RADIATIVE FLUX OUTWARDS DRIVEN BY CHANGING (DECREASING) P_{rad} (radiation pressure)

FOR A STELLAR ATMOSPHERE IN EQUILIBRIUM PROCESSES (ABS^{N}, EMISSION) BALANCE

- NO NET CHANGE IN ENERGY OF RADIATION FIELD

$\therefore F_{rad}$ CONSTANT AT EVERY LEVEL

$$= F_{SURFACE} = \sigma T_{eff}^4$$

and thus $\frac{dF_{rad}}{d\tau_\nu} = 0$ so that $\langle I \rangle = S$

Integrating $\frac{dP_{rad}}{d\tau_\nu} = \frac{1}{c} F_{rad} \Rightarrow P_{rad} = \frac{1}{c} F_{rad} \tau_\nu + \text{constant}$

$\rightarrow$ radiation pressure as function of τ_ν

CAN WE DERIVE TEMPERATURE STRUCTURE OF ATMOSPHERE?

ADOPT EDDINGTON APPROXIMATION

VERY SIMPLY $I_{OUT}(z) = I_{IN}(-z)$;

& AT TOP OF ATMOSPHERE $I_{IN} = 0$, $\tau_\nu = 0$

$[I_\nu(0,\theta) = B(T, \cos\theta)]$

$$\Rightarrow \langle I \rangle = \frac{1}{2}(I_{out} + I_{in}) = \frac{I_{OUT}}{2} \text{ at } z = 0, \tau_\nu = 0$$

$$\langle F_{rad} \rangle = \pi (I_{out} - I_{in}) = \pi I_{OUT} \text{ at } z = 0, \tau_\nu = 0$$

$$\langle P_{rad} \rangle = \frac{2\pi}{3c}(I_{out} + I_{in}) = \frac{4\pi}{3c}\langle I \rangle$$

earlier definitions indicate how these expressions obtained

WE HAVE $P_{rad} = \frac{1}{c} F_{rad} \tau_v + C$

$\therefore \frac{4\pi}{3c} \langle I \rangle = \frac{1}{c} F_{rad} \tau_v + C$

At top of atmosphere, $z = 0$, $\tau = 0$, $\langle I \rangle = \frac{I_{OUT}}{2}$

and $F_{rad} = \pi I_{OUT}$

$\therefore \langle I \rangle = F_{rad}/2\pi$

$\therefore \frac{4\pi}{3c} \frac{F_{rad}}{2\pi} = 0 + C \Rightarrow C = \frac{2}{3c} F_{rad}$

Substitute for C: $\therefore \frac{4\pi}{3c} \langle I \rangle = \frac{F_{rad}}{c} \tau_v + \frac{2 F_{rad}}{3c}$

$\therefore \frac{4\pi}{3} \langle I \rangle = F_{rad} (\tau_v + 2/3)$

Since $F = \sigma T_{eff}^4$, $\langle I \rangle = \frac{3}{4\pi} \sigma T_{eff}^4 (\tau_v + 2/3)$

IN LTE $S = B = \sigma T^4/\pi$ and $S = \langle I \rangle$

$\therefore \frac{\sigma T^4}{\pi} = \frac{3 \sigma T_{eff}^4}{4\pi} (\tau_v + 2/3)$

$\therefore T^4 = 3/4\, T_{eff}^4 (\tau_v + 2/3) \equiv$ variation of temp. with vertical optical depth in plane-parallel gray atmosphere

FOR $\tau_v = 2/3$, $T^4 = \frac{3}{4} T_{eff}^4 (4/3) = T_{eff}^4$

$\therefore$ STELLAR SURFACE (PHOTOSPHERE) AT $\tau_v = 2/3$

NOT AT $\tau = 0$

PHOTOSPHERE $\equiv$ AVERAGE DEPTH FROM WHICH PHOTONS ORIGINATE

i.e. we are seeing down to optical depth 2/3